CASA
Cyber Security in the Age of Large-Scale Adversaries

Decomposing Linear Layers

FSE 2023, (Kobe)

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Ideal vs Real World

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Open Design

- public specification
- public design rational
- public security results

Closed design

- no public specification
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Examples for closed designs: GEA-1, SIMON/SPECK, STREEBOG

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Example: Streebog [Russian standard]

- Russian hash function standard GOST R 34.11-2012
- SPN operating on a 64-byte state with
  - 8-bit permutation π (only given as a lookup-table)
  - permutation of bytes
  - 64×64 matrix with entries in $\mathbb{F}_2$, applied 8 times in parallel
- No design rationale or explanation how those were chosen

Reverse-engineering of building blocks

Research problem

Recover structure and/or design rationale

For S-boxes (well studied):

- very nice results for S-box π of Streebog [Perrin, ToSC 2019]

For linear layer: not well studied.

Our contribution

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Focus on the linear layer

Given a linear layer by a binary matrix, recover structure induced by constructions over extension fields.

- Problem 1: Given $M \in \mathrm{Mat}(\mathbb{F}_2, ms \times ms)$, decide whether $M \in \mathrm{Mat}(\mathbb{F}_{2^s}, m \times m)$
- Problem 2: Obfuscated case of Problem 1
- Problem 3: Decide whether M follows a well-known construction for MDS.

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1. Field Structure

2. Obfuscation and More Structure

Example: A matrix $M \in \mathrm{Mat}(\mathbb{F}_2, 12 \times 12)$

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$$
\begin{bmatrix}
1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
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0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1
\end{bmatrix}
$$

Example: A matrix $M \in \mathrm{Mat}(\mathbb{F}_2, 12 \times 12)$

Consider $s = 4$:

$$\left[\begin{array}{cccc|cccc|cccc}
1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
\hline
1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\
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0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\
\hline
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0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1
\end{array}\right] \stackrel{?}{=} \begin{bmatrix} M_{1,1} & M_{1,2} & M_{1,3} \\ M_{2,1} & M_{2,2} & M_{2,3} \\ M_{3,1} & M_{3,2} & M_{3,3} \end{bmatrix}$$

$M_{i,j} = \gamma^{N(i,j)}$ for $\gamma \in \mathbb{F}_{2^4}^*$

Usual field representation

$$
\left[\begin{array}{cccc|cccc|cccc}
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1&1&1&0&1&0&1&0&0&1&1&0\\
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1&1&1&0&1&0&0&1&1&0&0&0\\
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\end{array}\right]
\stackrel{?}{=}
\begin{bmatrix}
\gamma^{N(1,1)} & \gamma^{N(1,2)} & \gamma^{N(1,3)}\\
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- Usually, $\mathbb{F}_{2^s} \sim \mathbb{F}_2[X]/(f)$, where f is an irred. polynomial in $\mathbb{F}_2[X]$ of deg. s
- More general: $\mathbb{F}_{2^s}$ as a subring of $\mathrm{Mat}(\mathbb{F}_2, s \times s)$ (e.g., [Lidl, Niederreiter,'94])

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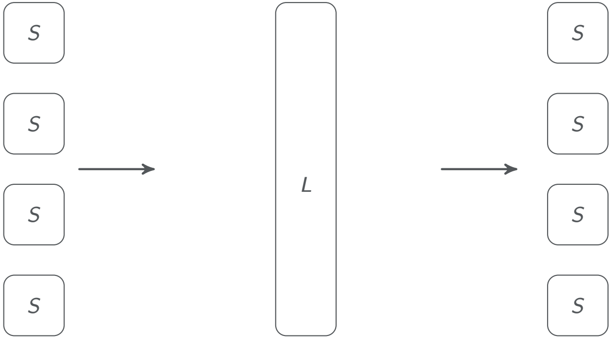
Choice



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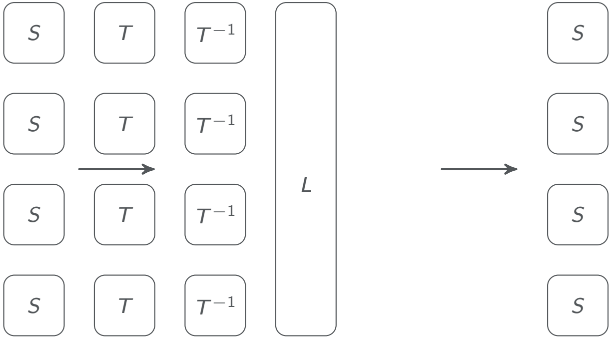

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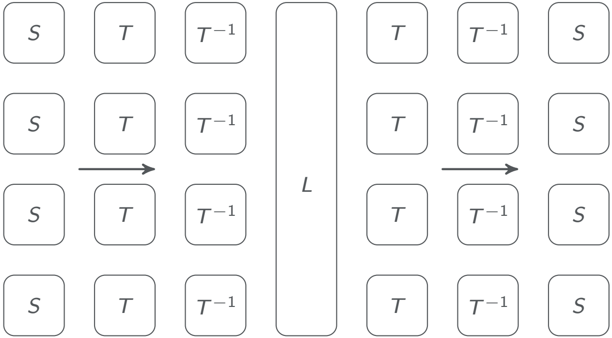

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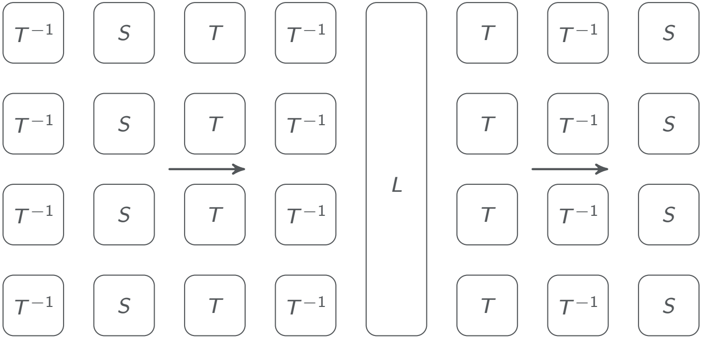

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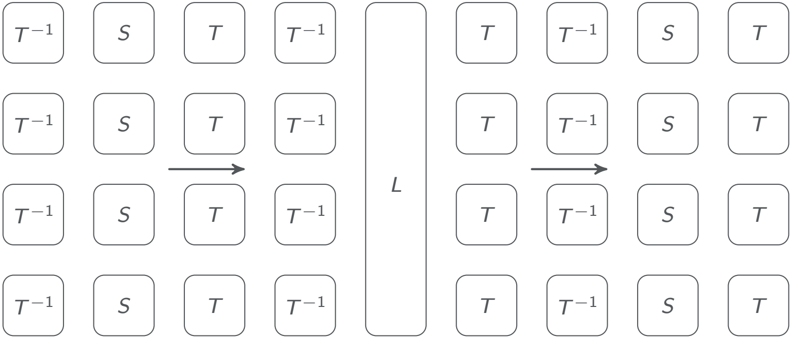

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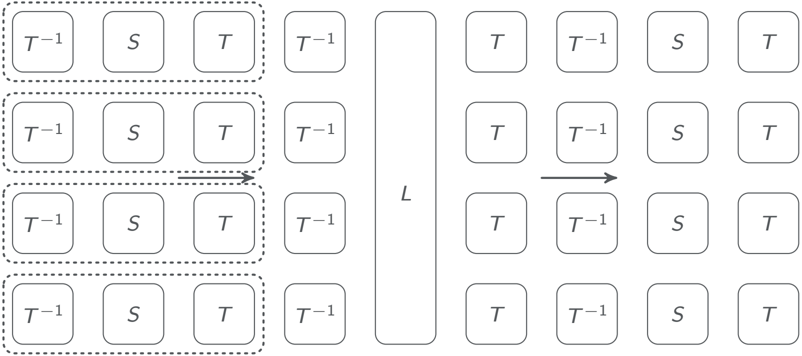

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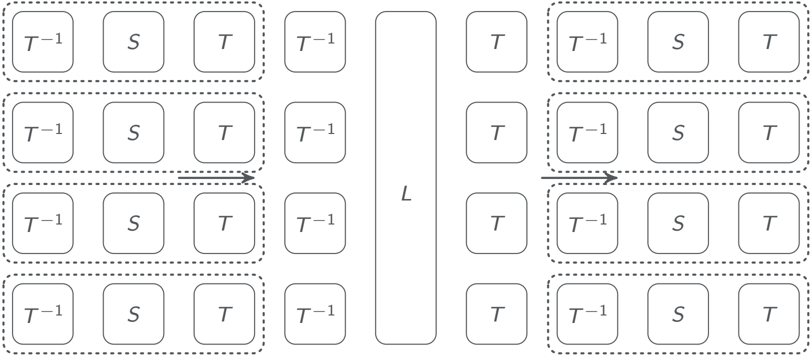

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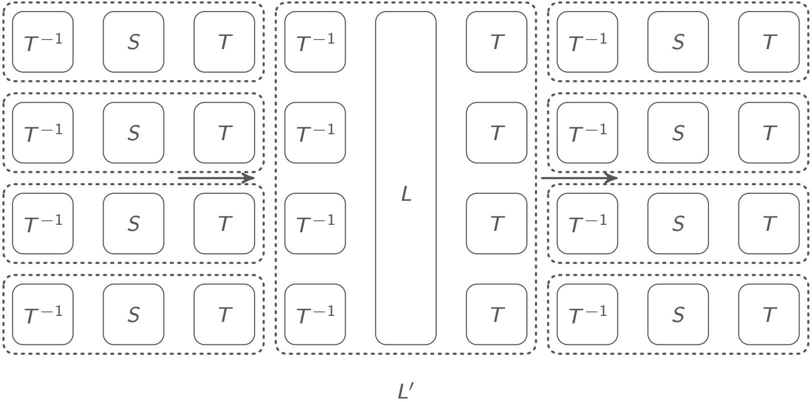

Matrix representations of $\mathbb{F}_{2^s}$

Starting with $L = (\gamma_{i,j})_{i,j}$

$$L' = \left(T\gamma_{i,j}T^{-1}\right)_{i,j}$$

The sub-matrices are still field elements, e.g.

$$\left(T\alpha T^{-1}\right)\left(T\beta T^{-1}\right) = \left(T\alpha\beta T^{-1}\right)$$

Problem (abstracted)

Let $\mathcal{S} = \{A_1, A_2, \ldots, A_n\} \subseteq \mathrm{GL}(s, \mathbb{F}_2)$. Decide whether the matrix ring $\mathbb{F}_2[\mathcal{S}]$ (i.e., the smallest subring of $\mathrm{Mat}(\mathbb{F}_2, s \times s)$ containing $\mathcal{S}$) is a field isomorphic to a (sub)field of $\mathbb{F}_{2^s}$.

$$\mathcal{S} = \{L'_{i,j}\}$$

When is $\mathbb{F}_2[\mathcal{S}]$ a field?

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Let's start easy: $\mathcal{S} = \{A\}$

Lemma (One Element in $\mathcal{S}$)

For $A \in \mathrm{GL}(s, \mathbb{F}_2)$, the ring $\mathbb{F}_2[A] = \{\sum_{i=0}^{m} c_i A^i \mid c_i \in \mathbb{F}_2, m \geq 0\}$ is a field isomorphic to a subfield of $\mathbb{F}_{2^s}$ if and only if the minimal polynomial of A, denoted m_A, is irreducible.

When is $\mathbb{F}_2[\mathcal{S}]$ a field? (cont.)

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Lemma (Multiple Elements in $\mathcal{S}$)

Let $\mathcal{S} = \{A_1, A_2, \ldots, A_n\} \subseteq \mathrm{GL}(s, \mathbb{F}_2)$. Then, $\mathbb{F}_2[\mathcal{S}]$ is a field isomorphic to a subfield of $\mathbb{F}_{2^s}$ if and only if the multiplicative subgroup $\langle \mathcal{S} \rangle$ of $\mathrm{GL}(s, \mathbb{F}_2)$ is cyclic and generated by an element with irreducible minimal polynomial.

Proof:

- $\Leftarrow$ is clear from the previous lemma
- If $\mathbb{F}_2[\mathcal{S}]$ is a field, its multiplicative group is cyclic, hence $\langle \mathcal{S} \rangle$ is a cyclic subgroup. Let α be a generator of $\langle \mathcal{S} \rangle$. We have $\mathbb{F}_2[\alpha] = \mathbb{F}_2[\mathcal{S}]$. By the previous lemma, m_α is irreducible.

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A group-theoretic question

Problem (further abstracted)

Decide whether a group $\langle A_1, A_2, \ldots, A_n \rangle \subseteq \mathrm{GL}(s, \mathbb{F}_2)$ is cyclic and generated by an element with irrreducible minimal polynomial.

Lemma (Folklore)

Let $G = \langle A_1, A_2 \rangle$ be a cyclic group

$$k_1 = \mathrm{ord}(A_1) \text{ and } k_2 = \mathrm{ord}(A_2).$$

Let h_1, h_2 be coprime positive integers such that $h_1 h_2 = \mathrm{lcm}(k_1, k_2)$ and, for $i \in \{1, 2\}$, $h_i \mid k_i$. Then, $G = \langle A_1^{k_1/h_1} \cdot A_2^{k_2/h_2} \rangle$.

- Applying this lemma iteratively allows to find a generator of a cyclic group $\langle A_1, \ldots, A_n \rangle$.

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Algorithm for deciding whether $\mathbb{F}_2[A_1, \dots, A_n]$ is a field

1. Apply iteratively to $G = \langle A_1, \dots, A_n \rangle$ to find a pseudo-generator α of G (α is indeed a generator if and only if G is cyclic)
2. We need to check whether $A_1, \dots, A_n$ are indeed elements of $\mathbb{F}_2[\alpha]$:
   For each A_i, check whether $A_i \in \mathrm{Span}(1, \alpha, \alpha^2, \dots, \alpha^{s-1})$.
   - If not, return "Not a field"
3. If m_α is not irreducible return "Not a field"
4. return α

Complexity

If we know the prime factorization of $2^s - 1$ running time is polynomial.

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Application to Streebog

For the linear layer of Streebog (element $M \in \mathrm{Mat}(\mathbb{F}_2, 64 \times 64)$), the running time is less than a second and we directly obtain

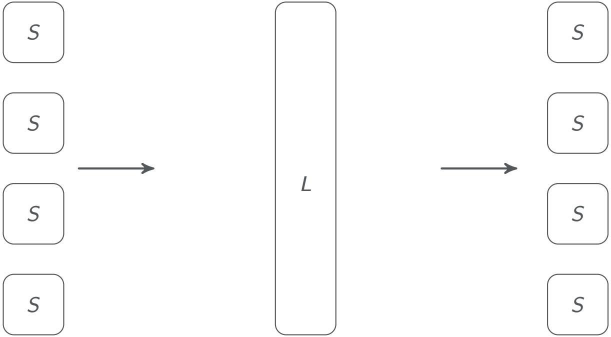
$$M = \begin{bmatrix} \gamma^{1} & \gamma^{64} & \gamma^{66} & \gamma^{39} & \gamma^{133} & \gamma^{249} & \gamma^{94} & \gamma^{135} \\ \gamma^{249} & \gamma^{84} & \gamma^{150} & \gamma^{0} & \gamma^{210} & \gamma^{1} & \gamma^{221} & \gamma^{32} \\ \gamma^{100} & \gamma^{16} & \gamma^{155} & \gamma^{15} & \gamma^{167} & \gamma^{36} & \gamma^{182} & \gamma^{57} \\ \gamma^{220} & \gamma^{174} & \gamma^{246} & \gamma^{217} & \gamma^{216} & \gamma^{17} & \gamma^{90} & \gamma^{198} \\ \gamma^{116} & \gamma^{188} & \gamma^{217} & \gamma^{246} & \gamma^{124} & \gamma^{127} & \gamma^{237} & \gamma^{206} \\ \gamma^{37} & \gamma^{129} & \gamma^{147} & \gamma^{243} & \gamma^{36} & \gamma^{167} & \gamma^{154} & \gamma^{89} \\ \gamma^{77} & \gamma^{66} & \gamma^{64} & \gamma^{238} & \gamma^{206} & \gamma^{3} & \gamma^{136} & \gamma^{124} \\ \gamma^{135} & \gamma^{230} & \gamma^{73} & \gamma^{137} & \gamma^{164} & \gamma^{32} & \gamma^{134} & \gamma^{1} \end{bmatrix}, \quad (1)$$

where

$$\gamma = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

1. Field Structure
2. Obfuscation and More Structure

(Heavy) Obfuscated Case: Even More Choice



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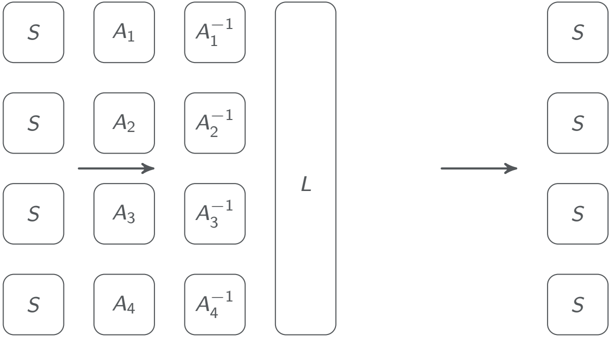

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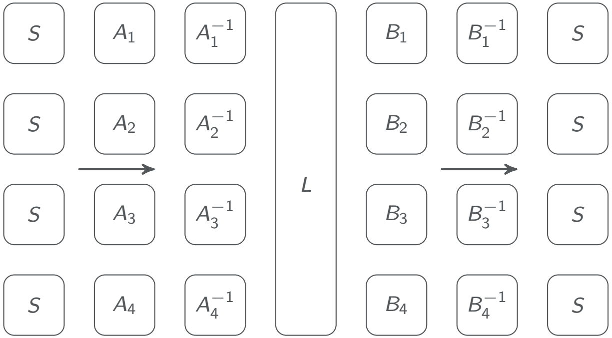

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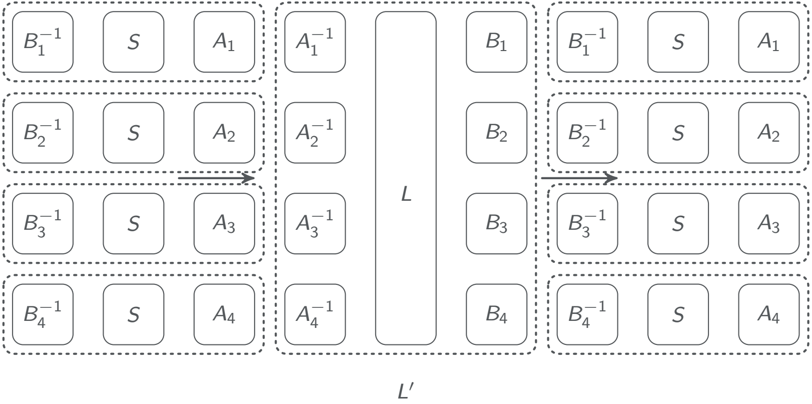
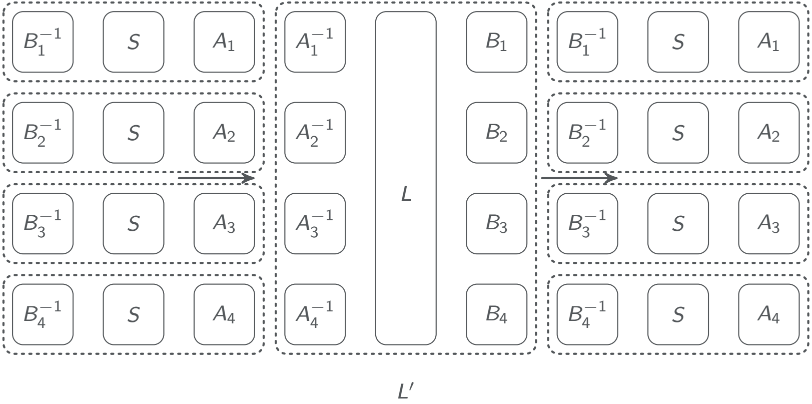


(Heavy) Obfuscated Case: Even More Choice



In the paper: Algorithm to efficiently recover L for L' also in this case

Cauchy MDS

CASA
Cyber Security in the Age of Large-Scale Adversaries

After we identified the field strucure: What more can be said?

Cauchy construction

We give a way to test if a (heavily obfuscated) matrix corresponds to a Cauchy-MDS Matrix.

Details in the paper.

Application to Streebog

Cyber Security in the Age of Large-Scale Adversaries

$$M = \begin{bmatrix} \gamma^{1} & \gamma^{64} & \gamma^{66} & \gamma^{39} & \gamma^{133} & \gamma^{249} & \gamma^{94} & \gamma^{135} \\ \gamma^{249} & \gamma^{84} & \gamma^{150} & \gamma^{0} & \gamma^{210} & \gamma^{1} & \gamma^{221} & \gamma^{32} \\ \gamma^{100} & \gamma^{16} & \gamma^{155} & \gamma^{15} & \gamma^{167} & \gamma^{36} & \gamma^{182} & \gamma^{57} \\ \gamma^{220} & \gamma^{174} & \gamma^{246} & \gamma^{217} & \gamma^{216} & \gamma^{17} & \gamma^{90} & \gamma^{198} \\ \gamma^{116} & \gamma^{188} & \gamma^{217} & \gamma^{246} & \gamma^{124} & \gamma^{127} & \gamma^{237} & \gamma^{206} \\ \gamma^{37} & \gamma^{129} & \gamma^{147} & \gamma^{243} & \gamma^{36} & \gamma^{167} & \gamma^{154} & \gamma^{89} \\ \gamma^{77} & \gamma^{66} & \gamma^{64} & \gamma^{238} & \gamma^{206} & \gamma^{3} & \gamma^{136} & \gamma^{124} \\ \gamma^{135} & \gamma^{230} & \gamma^{73} & \gamma^{137} & \gamma^{164} & \gamma^{32} & \gamma^{134} & \gamma^{1} \end{bmatrix},$$

$$\gamma = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix},$$

is a Cauchy matrix with

$$(u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8) = (\gamma^{120}, \gamma^{223}, \gamma^{198}, \gamma^{57}, \gamma^{49}, \gamma^{166}, \gamma^{131}, \gamma^{254})$$

$$(v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8) = (\gamma^{77}, \gamma^{82}, \gamma^{59}, \gamma^{220}, \gamma^{72}, \gamma^{209}, \gamma^{4}, 0).$$

The End

Thank you very much for your attention!
Questions?