Vectorized linear approximations for attacks on SNOW 3G

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Outline

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Confidentiality and Integrity Protection in Cellular Networks

- Three standardized algorithms in LTE: SNOW 3G, AES, ZUC
  - 128-bit security level

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Confidentiality and Integrity Protection in Cellular Networks

- Three standardized algorithms in LTE: SNOW 3G, AES, ZUC
  - 128-bit security level
- 5G: 256-bit security algorithms
- One possible solution: reuse existing algorithms
  - Security under the 256-bit key length should be investigated
- **Contribution**: give linear cryptanalysis of SNOW 3G
  - Distinguishing attack 2^{172}
  - Correlation attack 2^{177}

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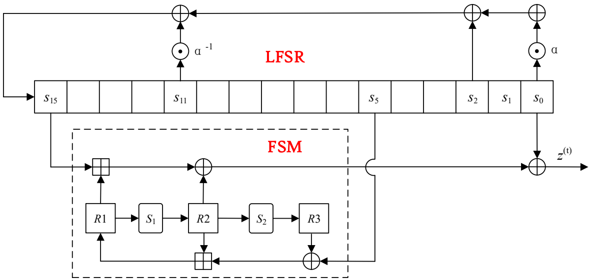
- A stream cipher with a linear part and a non-linear part



- **Linear part:** linear feedback shift register (LFSR)
- **Non-linear part**: finite state machine (FSM)

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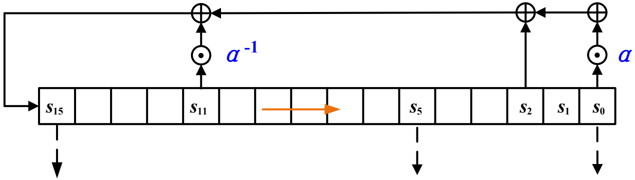
LFSR in SNOW 3G



- Defined over $GF(2^{32})$, 16 cells $\times$ 32 bits / cell $=$ 512 bits

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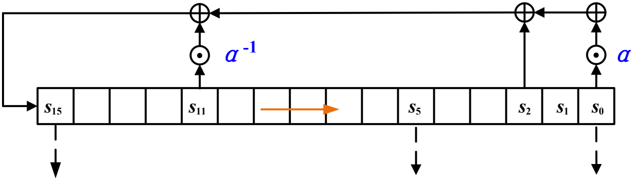

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- **Feedback polynomial**:

$$P(x) = \alpha x^{16} + x^{14} + \alpha^{-1} x^5 + 1 \in GF(2^{32})[x]$$

  - α is a root of a polynomial in $GF(2^8)[x]$

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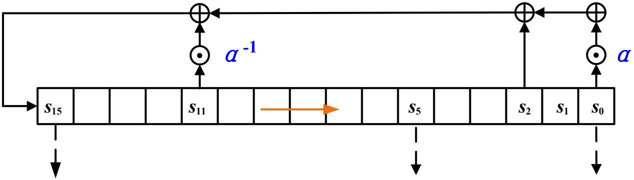

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- **LFSR update**:

$$s_i^{(t+1)} = s_{i+1}^{(t)} \quad (0 \le i \le 14),$$
$$s_{15}^{(t+1)} = \alpha^{-1} s_{11}^{(t)} + s_2^{(t)} + \alpha s_0^{(t)}.$$

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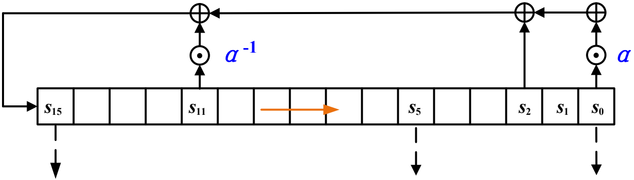

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- $s_{15}^{(t)}, s_5^{(t)}, s_0^{(t)}$ used to update FSM and generate keystream

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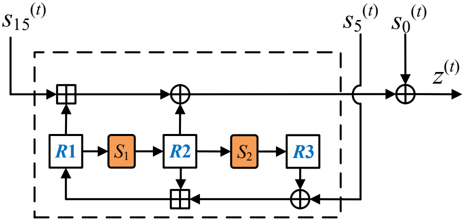
FSM in SNOW 3G



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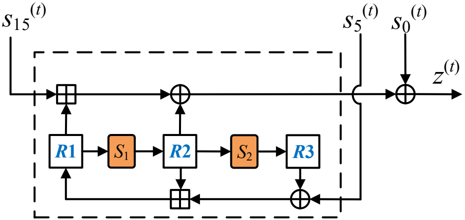

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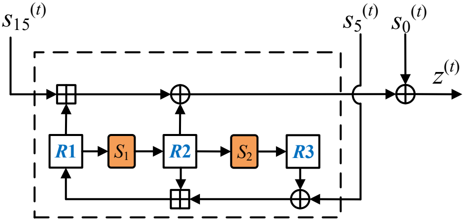

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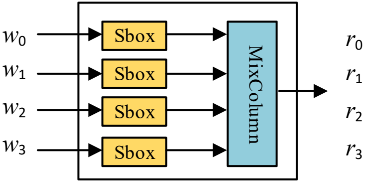
$$
\begin{aligned}
R1^{(t+1)} &= R2^{(t)} \boxplus_{32} (R3^{(t)} \oplus s_5^{(t)}) \\
R2^{(t+1)} &= S_1(R1^{(t)}) \\
R3^{(t+1)} &= S_2(R2^{(t)})
\end{aligned}
$$

- S_1, S_2 are 32-to-32 S-transforms

S-transforms in FSM



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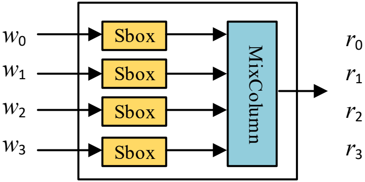

- $S_1 = L_1 \cdot S_R$, S_R is the AES S-box

$$\begin{pmatrix} r_0 \\ r_1 \\ r_2 \\ r_3 \end{pmatrix} = \begin{pmatrix} x & x+1 & 1 & 1 \\ 1 & x & x+1 & 1 \\ 1 & 1 & x & x+1 \\ x+1 & 1 & 1 & x \end{pmatrix} \cdot \begin{pmatrix} S_R(w_0) \\ S_R(w_1) \\ S_R(w_2) \\ S_R(w_3) \end{pmatrix}$$

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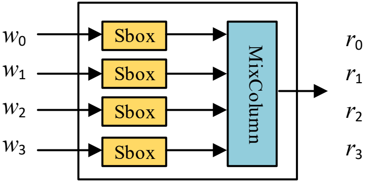

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- $S_2 = L_2 \cdot S_Q$, S_Q is derived from the Dickson polynomials

$$\begin{pmatrix} r_0 \\ r_1 \\ r_2 \\ r_3 \end{pmatrix} = \begin{pmatrix} y & y+1 & 1 & 1 \\ 1 & y & y+1 & 1 \\ 1 & 1 & y & y+1 \\ y+1 & 1 & 1 & y \end{pmatrix} \cdot \begin{pmatrix} S_Q(w_0) \\ S_Q(w_1) \\ S_Q(w_2) \\ S_Q(w_3) \end{pmatrix}$$

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- **Basic Idea**: approximate non-linear components as linear ones, further derive some linear relationships, involving:

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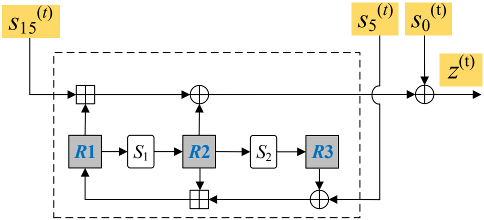
- **Key Point**: to find a good approximation with a large bias

Linear Approximation of FSM in SNOW 3G



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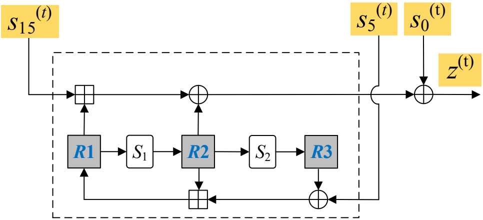

- Explore linear expression including only s_{15}, s_5, s_0, z

$$\bigoplus_{i \in I} \left(c_z^{(t+i)} z^{(t+i)} \oplus c_{15}^{(t+i)} s_{15}^{(t+i)} \oplus c_5^{(t+i)} s_5^{(t+i)} \oplus c_0^{(t+i)} s_0^{(t+i)} \right)$$

- $c_z^{(t+i)}, c_{15}^{(t+i)}, c_5^{(t+i)}, c_0^{(t+i)}$ are linear masking matrices

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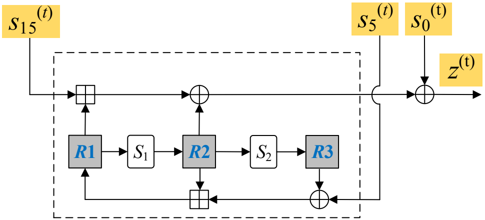

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  - $c_z^{(t+i)}, c_{15}^{(t+i)}, c_5^{(t+i)}, c_0^{(t+i)}$ are linear masking matrices
- The SEI of it evaluates the quality of the approximation
  - Find good time set I and masking matrices

Linear Approximation of FSM

Consider 3 consecutive keystream blocks to cancel out R_1, R_2, R_3

Registers update and recursion at three time instances

$$R2^{(t+1)} = L_1 \cdot S_R(R1^{(t)}) \qquad R1^{(t-1)} = S_R^{-1} \cdot L_1^{-1}(R2^{(t)})$$
$$R3^{(t+1)} = L_2 \cdot S_Q(R2^{(t)}) \qquad R2^{(t-1)} = S_Q^{-1} \cdot L_2^{-1}(R3^{(t)})$$
$$R1^{(t+1)} = R2^{(t)} \boxplus_{32} (R3^{(t)} \oplus s_5^{(t)})$$

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$$R1^{(t-1)} = S_R^{-1} \cdot L_1^{-1}(R2^{(t)})$$
$$R2^{(t-1)} = S_Q^{-1} \cdot L_2^{-1}(R3^{(t)})$$

Keystream symbols at 3 consecutive time instances

$$z^{(t-1)} = (S_R^{-1} L_1^{-1}(R2^{(t)}) \boxplus s_{15}^{(t-1)}) \oplus S_Q^{-1} L_2^{-1}(R3^{(t)}) \oplus s_0^{(t-1)}$$
$$z^{(t)} = (R1^{(t)} \boxplus s_{15}^{(t)}) \oplus R2^{(t)} \oplus s_0^{(t)}$$
$$L_1^{-1} z^{(t+1)} = L_1^{-1}(R2^{(t)} \boxplus (R3^{(t)} \oplus s_5^{(t)}) \boxplus s_{15}^{(t+1)}) \oplus S_R(R1^{(t)}) \oplus L_1^{-1} s_0^{(t+1)}$$

L_1^{-1} is the inverse of L_1, used as a linear masking matrix

24-bit Linear Approximation

Build 24-bit symbols: combining the first bytes

$$\underbrace{\begin{pmatrix} z^{(t-1)} \\ z^{(t)} \\ L_1^{-1} z^{(t+1)} \end{pmatrix}_{[0,0,0]}}_{\text{Sample at } (t):\ Z^{(t)}} = \underbrace{\begin{pmatrix} (S_R^{-1}(L_1^{-1} R2^{(t)}) \boxplus s_{15}^{(t-1)}) \oplus s_{15}^{(t-1)} \oplus S_Q^{-1}(L_2^{-1} R3^{(t)}) \\ R2^{(t)} \\ L_1^{-1}[(R2^{(t)} \boxplus (R3^{(t)} \oplus s_5^{(t)}) \boxplus s_{15}^{(t+1)}) \oplus s_5^{(t)} \oplus s_{15}^{(t+1)}] \end{pmatrix}_{[0,0,0]}}_{\text{24-bit Noise } N2^{(t)}}$$

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  - $\epsilon(N1^{(t)})$: loop over $R1^{(t)}[0], s_{15}^{(t)}[0]$ in $O(2^{16})$

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  - $\epsilon(N1^{(t)})$: loop over $R1^{(t)}[0], s_{15}^{(t)}[0]$ in $O(2^{16})$
  - How about $\epsilon(N2^{(t)})$? (4 32-bit variables: $R2, R3, s_5, s_{15}$)

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Bias Computing of $\epsilon(N2^{(t)})$

Split variables / noise expression into smaller fields [ZXM15][1][MJ05] [2]

- Compute sub-distributions and combine them

[1] Zhang B., et al. Fast correlation attacks over extension fields, large-unit...CRYPTO'2015.
[2] Maximov A, et al. Fast computation of large distributions and... ASIACRYPT 2005.

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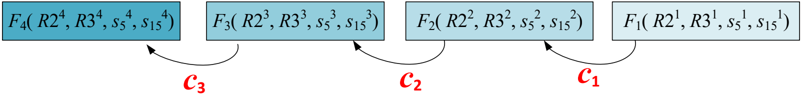

- Consider carries between different bytes

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- Decision rule:

$$P_X = \begin{cases} P_N, & \text{if } D(P_X||P_U) > D(P_X||P_N), \\ P_U, & \text{if } D(P_X||P_U) < D(P_X||P_N). \end{cases}$$

  - $D(x||y)$: KL divergence (or relative entropy) between x, y
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- Build random sequences: lower 24 bits of keystream symbols
- For every sequence, check which distribution it follows
- Errors:
  - TYPE I: a noise distribution is judged as random
  - TYPE II: a random distribution is judged as biased

Results of the Experimental Verification

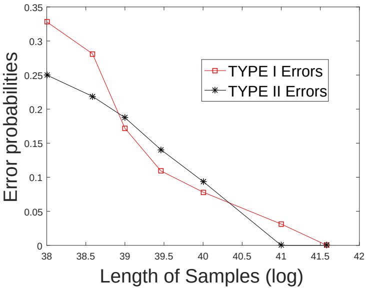

Figure. Error probabilities under different lengths of samples

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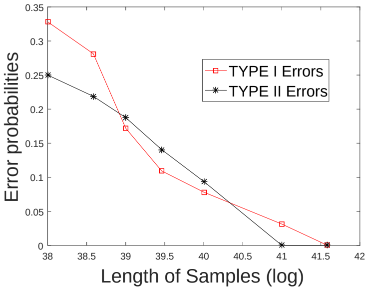

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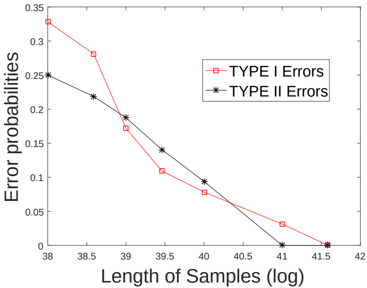

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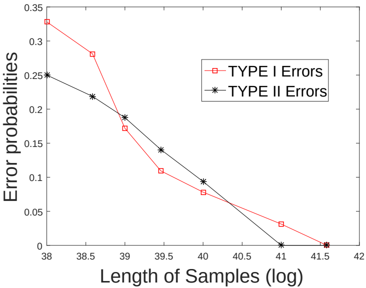

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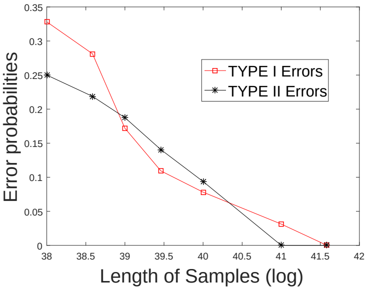

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- The bias should be correct!

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- Equivalent to finding a multiple of the generating polynomial $P(x)$ of weight 3, 4, or 5, with all coefficients being 1

Finalize the Distinguishing Attack

- Find a weight-4 multiple $K(x)$ using method from [LJ14] [3]
  - Time and storage complexities $O(2^{172})$

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- Bias: $\epsilon(X) = \epsilon(4 \times N) > 2^{-163}$ (regarded as independent)
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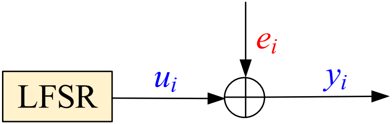

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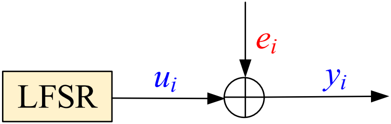

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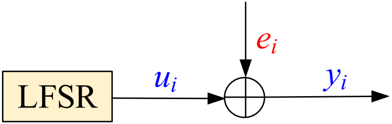

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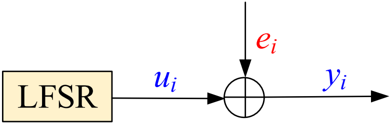

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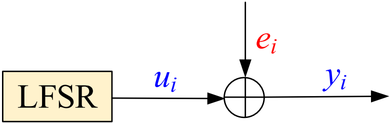

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- When $R = \log(2^n) \cdot l/N < C$: can be successfully decoded

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- The codeword and received codeword symbols:

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- Recover $\mathbf{s}$ according to the y sequence
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Finalize the Correlation Attack [ZXM15][4]

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  - Generating parity checks involving fewer LFSR states
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  - Distinguishing attack with complexity 2^{172}
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- If the key length in SNOW 3G would be increased to 256 bits, there are academic attacks on it

Conclusions

- We give linear cryptanalysis of SNOW 3G
  - 24-bit linear approximation of bias 2^{-37}
  - Verified the bias by collecting a large number of samples
  - Distinguishing attack with complexity 2^{172}
  - Correlation attack with complexity 2^{177}
- If the key length in SNOW 3G would be increased to 256 bits, there are academic attacks on it
- Not an immediate threat for 5G.

Thank you

Thank you for your attention!