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New Conditional Cube Attack on Keccak Keyed Modes

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Abstract. The conditional cube attack on round-reduced Keccak keyed modes was proposed by Huang *et al.* at EUROCRYPT 2017. In their attack, a conditional cube variable was introduced, whose diffusion was significantly reduced by certain key bit conditions. The attack requires a set of cube variables which are not multiplied in the first round while the conditional cube variable is not multiplied with other cube variables (called ordinary cube variables) in the first two rounds. This has an impact on the degree of the output of Keccak and hence gives a distinguisher. Later, the MILP method was applied to find ordinary cube variables. However, for some Keccak based versions with few degrees of freedom, one could not find enough ordinary cube variables, which weakens or even invalidates the conditional cube attack.

In this paper, a new conditional cube attack on Keccak is proposed. We remove the limitation that no cube variables multiply with each other in the first round. As a result, some quadratic terms may appear in the first round. We make use of some new bit conditions to prevent the quadratic terms from multiplying with other cube variables in the second round, so that there will be no cubic terms in the first two rounds. Furthermore, we introduce the *kernel quadratic term* and construct a *6-2-2 pattern* to reduce the diffusion of quadratic terms significantly, where the θ operation even in the second round becomes an identity transformation (CP-kernel property) for the *kernel quadratic term*. Previous conditional cube attacks on Keccak only explored the CP-kernel property of θ operation in the first round. Therefore, more degrees of freedom are available for ordinary cube variables and fewer bit conditions are used to remove the cubic terms in the second round, which plays a key role in the conditional cube attack on versions with very few degrees of freedom. We also use the MILP method in the search of cube variables and give key-recovery attacks on round-reduced Keccak keyed modes.

As a result, we reduce the time complexity of key-recovery attacks on 7-round Keccak-MAC-512 and 7-round Ketje Sr v2 from 2^{111}, 2^{99} to 2^{72}, 2^{77}, respectively. Additionally, we have reduced the time complexity of attacks on 9-round `KMAC256` and 7-round Ketje Sr v1. Besides, practical attacks on 6-round Ketje Sr v1 and v2 are also given in this paper for the first time.

Keywords: Conditional Cube Attack, Keccak, `KMAC`, Ketje, MILP

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1 Introduction

Keccak [BDPVA09], designed by Bertoni *et al.*, has been selected as the new cryptographic hash function standard SHA-3. As one of the most important cryptographic standards, Keccak attracts a lot of attention from world wide researchers and engineers. Till now, many cryptanalysis results [BCC11, DDS12, DDS13, DGPW12, GLS16, JN15, MPS13] and evaluation tools [DA12, DEM15, MDA17] have been proposed, including the recent impressive collision attacks [QSLG17, SLG17].

Besides, some important keyed constructions based on Keccak-*p* permutations have been proposed, including Keccak-MAC [BDPA11], KMAC [KCP16], Keyak [BDP+16b], Ketje [BDP+16a] and Kravatte [BDH+17]. At EUROCRYPT 2015, Dinur *et al.* [DMP+15] for the first time considered the security of the Keccak keyed modes utilizing cube-attack-like cryptanalysis and give some key-recovery attacks on reduced-round Keccak-MAC and Keyak. At CT-RSA 2015, Dobraunig *et al.* [DEMS15] evaluated the security of Ascon [DEMS16] against cube-attack-like cryptanalysis. Later, this method was improved by Dong *et al.* [DLWQ17], Bi *et al.* [BDL+18] and Song *et al.* [SG18].

Conditional cube attacks on round-reduced Keccak keyed modes were proposed by Huang *et al.* [HWX+17] at EUROCRYPT 2017. Firstly they select a conditional cube variable. Bit conditions are added to reduce the diffusion of the conditional cube variable significantly. Thus, they could find a set of cube variables that are not multiplied in the first round, while the conditional cube variable is not multiplied with other cube variables (called ordinary cube variables) in the first two rounds. The key bit conditions lead to a key-recovery attack. Later, Li *et al.* [LDW17] applied conditional cube attacks on Ascon. At ASIACRYPT 2017, Li *et al.* for the first time introduced a MILP model based method to improve the conditional cube attack, [LBDW17]. Later, the MILP model was improved by Song *et al.* [SGSL18] at ASIACRYPT 2018. However, as shown in previous works [LBDW17, SGSL18, DLWQ17], for some Keccak based versions with very few degrees of freedom, one could not find enough ordinary cube variables, which weakens or even invalidates the conditional cube attack.

In this paper, we put forward a new conditional cube attack on Keccak keyed modes. We use the 7-round attack on Keccak to explain our new ideas:

1. We remove the limitation that no cube variables multiply with each other in the first round. In Huang *et al.*'s attack [HWX+17], the selected 64 cube variables must not be multiplied together in the first round. Therefore, the expected degree of the output of the 7-round Keccak-*p* permutation is $2^6 = 64$ (note that the degree of Keccak round function is 2). However, they find a conditional cube variable that does not multiply with other cube variables under suitable bit conditions. So, the expected degree under correct conditions will be 63, rather than 64. This gives the distinguisher.

   In our new attack, different from previous conditional cube attacks, we select 65 cube variables, two of which will be multiplied together to generate quadratic terms in the first round. Therefore, the degree of the output of the 7-round Keccak-*p* permutation will be $2^6 + 1 = 65$ (this will be proved in Section 4). However, by imposing some bit conditions, the quadratic terms of cube variables will not be multiplied with other cube variables in the 2nd round. Therefore, the degree of the output polynomials of the 2-round Keccak-*p* permutation is still 2. So, the expected output degree of the 7-round Keccak-*p* permutation round under correct conditions will be 64, rather than 65. This gives the new distinguisher.

2. In Huang *et al.*'s conditional cube attack [HWX+17], the so-called 2-2-22 pattern is used to select a conditional cube variable. In their work, the conditional cube variable v_0 is in the CP-kernel in the first round. Actually, only 2 bits of the initial state

contain v_0, and they are in the same column. Therefore θ in the first round becomes an identity transformation for v_0. With correct bit conditions, v_0 still occupies 2 bit positions in the output state of χ in the first round. After θ in the second round, the positions of v_0 will spread to 22 bits.

Then they select ordinary cube variables that do not multiply together in the first round, and do not multiply with v_0 in the first two rounds. As shown in previous works [LBDW17, SGSL18, DLWQ17], it is hard to find enough ordinary cube variables that do not multiply with v_0 in the first two rounds for Keccak versions with few degrees of freedom. A natural idea to improve the previous works is to reduce the bit positions occupied by v_0. In [LBDW17, BLD+18], the 6-6-6 pattern is used to select v_0. However, for Keccak-MAC-512 and Ketje Sr, no 6-6-6 patterns exist (the 6-6-6 pattern's bit positions are occupied by the key bits or padding bits and could not be selected as variables).

In our situation, we only care the quadratic terms (e.g. v_0v_1) of cube variables should not be multiplied with other cube variables in the second round. So what we have to do is to reduce the diffusion of v_0v_1.

3. In order to reduce the diffusion of v_0v_1 further, we introduce the so-called *kernel quadratic term*, which follows a new *6-2-2 pattern*. In our attack, v_0v_1 appears in only 2 bit positions in the output of the first round. Furthermore, the 2 bit positions are in the same column and follow the CP-kernel property even in the θ operation of the second round. Therefore, the number of bit positions of v_0v_1 is still 2 before the χ operation in the second round. Thus, we can find enough other cube variables that do not multiply with v_0v_1 in the second round with ease and can perform new key-recovery attacks on versions with few degrees of freedom, like Keccak-MAC-512 and Ketje Sr v2.

The results are summarized in Table 1. Based on our new conditional cube attack, the time complexity of key-recovery attacks on 7-round Keccak-MAC-512, 7-round Ketje Sr v2, and 9-round KMAC256 is reduced from 2^{111}, 2^{99} and 2^{147} to 2^{72}, 2^{77} and 2^{139}, respectively. We also give the first practical attacks on 6-round Ketje Sr v1 and v2.

Organization. Section 2 gives some notation, followed by a brief description of Keccak-MAC, Ketje and KMAC. In Section 3, we briefly describe related works. The model of the new conditional cube attack is introduced in Section 4, and its applications to Keccak-MAC, Ketje and KMAC are provided in Section 5. At last, we conclude this paper in Section 6.

2 Preliminaries

This section gives the notations we use in the rest of the paper, and also describes the Keccak-p permutations, Keccak-MAC, Ketje and KMAC.

2.1 Notations

For convenience, we list the notations in the following.

2.2 The Keccak-p permutations

The Keccak-p permutations are derived from the Keccak-f permutations [BDPVA09] and have a tunable number of rounds. A Keccak-p permutation is denoted as Keccak-$p[b, n_r]$. Its width is $b = 25 \times \ell$, with $b \in \{25, 50, 100, 200, 400, 800, 1600\}$, and its number of rounds is n_r. As depicted in Figure 1, Keccak-$p[b, n_r]$ works on a state A of size b,

Table 1: Summary of key-recovery attacks on Keccak keyed modes

Variant	Key Size	Capacity	Method	Rounds	Time	Source
KMAC256	256	512	conditional†	9	2^{147}	[SGSL18]
			conditional	9	2^{139}	Subsection 5.3
Keccak-MAC	128	1024	conditional	6	2^{58}	[LBDW17]
			conditional	6	2^{40}	[SGSL18]
			cube-like	7	$2^{112.6}$	[BDL+18]
			cube-like	7	2^{111}	[SG18]
			conditional	7	2^{72}	Subsection 5.1
Ketje Sr v1	128	–	cube-like	6	2^{73}	[DLWQ17]
			conditional	6	$2^{40.6}$	Section 5.2.1
			cube-like	7	$2^{115.32}$	[DLWQ17]
			conditional	7	2^{91}	[SGSL18]
			conditional	7	2^{75}	Section 5.2.1
Ketje Sr v2	128	–	cube-like	6	$2^{65.6}$	[DLWQ17]
			conditional	6	$2^{40.6}$	Section 5.2.2
			cube-like	7	$2^{113.58}$	[DLWQ17]
			cube-like	7	2^{99}	[SG18]
			conditional	7	2^{77}	Section 5.2.2

†: in this table, "conditional" is short for conditional cube attack, and "cube-like" means cube-attack-like method.

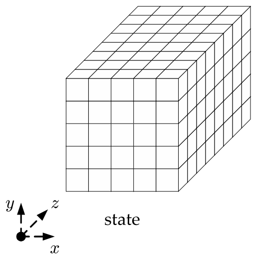
S_0	the initial state of the Keccak-p permutation,
$S_{i-1,\theta}$	the internal state after θ in the i-th round of Keccak-p, $i \geq 1$,
$S_{i-1,\pi}$	the internal state after π in the i-th round, $i \geq 1$,
S_i	the output state of the i-th round, $i \geq 1$,
	thus the internal states of the i-th round are as follows:
	$S_{i-1} \xrightarrow{\theta} S_{i-1,\theta} \xrightarrow{\rho} S_{i-1,\rho} \xrightarrow{\pi} S_{i-1,\pi} \xrightarrow{\chi} S_{i-1,\chi} \xrightarrow{\iota} S_i$.
ℓ	the word size of one lane,
$(*, y, z)$	the index of a row, $0 \leq y \leq 4$, $0 \leq z \leq \ell - 1$,
$(x, *, z)$	the index of a column, $0 \leq x \leq 4$, $0 \leq z \leq \ell - 1$,
$(x, y, *)$	the index of a lane, $0 \leq x, y \leq 4$,
(x, y, z)	the index of a bit, $0 \leq x, y \leq 4$, $0 \leq z \leq \ell - 1$,
$A[x][y]$	the lane indexed by $(x, y, *)$ of state A, $0 \leq x, y \leq 4$,
$A[x][y][z]$	the bit indexed by (x, y, z) of state A, $0 \leq x, y \leq 4$, $0 \leq z \leq \ell - 1$.

which can be represented as 5×5 $\frac{b}{25}$-bit lanes. $A[x][y]$ represents the lane indexed by $(x, y, *)$ of state A, $0 \leq x, y \leq 4$, and the computation of indices x and y is modulo 5. The round function R consists of five operations, denoted as $\mathtt{R} = \iota \circ \chi \circ \pi \circ \rho \circ \theta$, and the details are as follows:

$$
\begin{aligned}
&\theta : A[x][y] = A[x][y] \oplus \textstyle\sum_{j=0}^{4} (A[x-1][j] \oplus (A[x+1][j] \lll 1)).\\
&\rho : A[x][y] = A[x][y] \lll \rho[x, y].\\
&\pi : A[y][2x + 3y] = A[x][y].\\
&\chi : A[x][y] = A[x][y] \oplus ((\neg A[x+1][y]) \wedge A[x+2][y]).\\
&\iota : A[0][0] = A[0][0] \oplus RC.
\end{aligned}
$$

2.3 Keccak-MAC

Keccak is based on the sponge construction [BDPVA07], and its internal permutation is Keccak-$p[1600, 24]$. The sponge construction has two parameters, the capacity c and



0, 0	1, 0	2, 0	3, 0	4, 0
0, 1	1, 1	2, 1	3, 1	4, 1
0, 2	1, 2	2, 2	3, 2	4, 2
0, 3	1, 3	2, 3	3, 3	4, 3
0, 4	1, 4	2, 4	3, 4	4, 4

Figure 1: (a) The Keccak state [BDPVA09], (b) A slice of state A

the rate r with $c + r = 1600$. Keccak[c] denotes Keccak with capacity c. The state of Keccak is initialized to 0. The input is a variable-length message M and the output is a 128-bit digest. The message M is split into r-bit blocks. Keccak processes the blocks iteratively by absorbing them into the first r bits of the state and the permutation of Keccak-p[1600, 24]. Keccak-MAC [BDPA11] is a Keccak-based MAC, where the input is the concatenation of key and message. The key size is assumed to be 128 bits. Keccak-MAC-512 is a MAC based on Keccak[1024].

2.4 Ketje

Ketje [BDP+16a] is a submission to the CAESAR competition for authenticated encryption by the Keccak team. For Ketje v1, two instances are proposed, Ketje Sr and Jr with 400-bit and 200-bit state sizes, respectively. In the latest Ketje v2, another two instances Ketje Minor and Major are added to the family, with 800-bit and 1600-bit state sizes, respectively. Ketje Sr is the primary recommendation. The four concrete instances of Ketje v2 are shown in Table 2. In the following, we give a brief overview on Ketje v2. For a complete description, we refer to the design document [BDP+16a].

The structure of Ketje is an authenticated encryption mode MonkeyWrap, which is based on MonkeyDuplex [BDPA11]. It consists of four parts: initialization, processing associated data, processing the plaintext and finalization. Figure 2 illustrates the scheme of Ketje v2, where the finalization is omitted, f_0=Keccak-$p^*[b, 12]$ and f_1=Keccak-$p^*[b, 1]$. Ketje takes the following parameters: the key K, the *Nonce*, the associated data σ_0,...,σ_j and the plaintext M_0. When varying the value of *Nonce*, we can obtain the corresponding output $\mathrm{pad}(M_0) \oplus C_0$.

In Ketje v2, *the twisted permutations*, Keccak-$p^*[b]$=$\pi\circ$ Keccak-$p[b] \circ \pi^{-1}$, are introduced to effectively re-order the bits in the state. As shown in Figure 3, π^{-1} : $A[x + 3y][x] = A[x][y]$ is the inverse of π.

Table 2: Four instances of Ketje v2

Name	f	r	Main use case
Ketje Jr	Keccak-p^*[200]	16	lightweight
Ketje Sr	Keccak-p^*[400]	32	lightweight
Ketje Minor	Keccak-p^*[800]	128	lightweight
Ketje Major	Keccak-p^*[1600]	256	high performance

2.5 `KMAC`

`KMAC` (Keccak Message Authentication Code) [KCP16] is a keyed hash function, whose output length is variable. It has two variants: `KMAC128` and `KMAC256` with capacities set as 256 and 512 bits, respectively. They are based on Keccak[$c = 256$] and Keccak[$c = 512$].

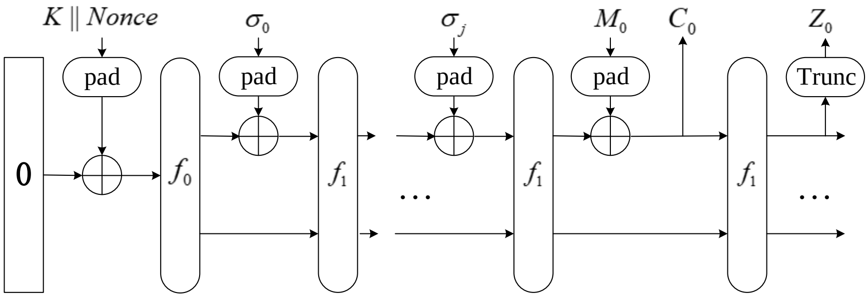


Figure 2: Ketje, where the finalization is omitted [SGSL18].

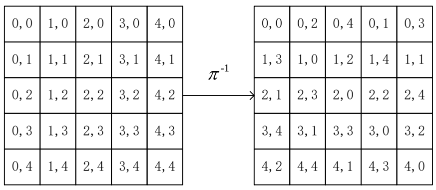


Figure 3: π^{-1}

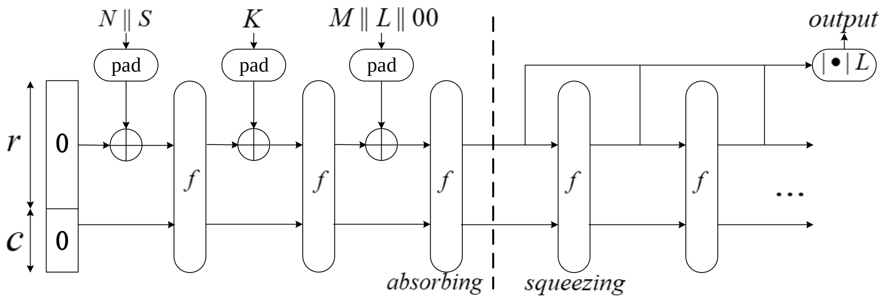


Figure 4: Construction of `KMAC`

`KMAC` takes the following parameters: the key K, the variable length message M, the output length L, the name string $N =$ "KMAC" and the optional customization bit string S of any length (including 0).

Figure 4 illustrates the processing of one message block. Actually, we treat the state before padding of the message as the initial state in our attack. Thus the structure is similar to the above two ciphers, and the whole secret 1600-bit state is treated as an equivalent of the key to be recovered. The output length of `KMAC` is variable, while that of `KMAC128` is no less than 256 bits and `KMAC256` is no less than 512 bits.

3 Related Work

3.1 Cube Attack

The cube attack [DS09] was introduced by Dinur and Shamir at EUROCRYPT 2009. In their work, an output bit of a symmetric cryptographic scheme is described as a master polynomial $p(k_1, ..., k_n, v_1, ..., v_m)$ over $\mathbb{F}_2$, which can be written as a sum of two polynomials:

$$p(k_1, ..., k_n, v_1, ..., v_m) = t_I \cdot p_{S(I)} + q(k_1, ..., k_n, v_1, ..., v_m),$$

where $k_1, ..., k_n$ are secret variables (e.g. the key bits), and $v_1, ..., v_m$ are public variables (e.g. the nonce or IV bits); t_I is called maxterm and is a product of certain public variables; $p_{S(I)}$ is called superpoly; $q(k_1, ..., k_n, v_1, ..., v_m)$ is the remainder polynomial and none of the terms in it is divisible by t_I. The maxterm t_I is defined through a subset of indices I, called a cube. In order to get the super polynomial $p_{S(I)}$, one assigns all possible values to the variables contained in I, evaluates the master polynomial and sums up the results [DS09].

3.2 Dynamic Cube Attack

The dynamic cube attack [DS11] was first introduced to analyse Grain-128 by Dinur and Shamir at FSE 2011. The basic idea is to simplify a complex polynomial $p(k_1, ..., k_n, v_1, ..., v_m)$: $p(k_1, ..., k_n, v_1, ..., v_m) = P_1 \times P_2 + P_3$ where the degree of P_3 is lower than that of $p(k_1, ..., k_n, v_1, ..., v_m)$, and P_1 contains a linear public term called a dynamic variable. A dynamic variable is a variable assigned with a function in some key bits and cube variables, and is used to zeroize P_1. Thus, $p(k_1, ..., k_n, v_1, ..., v_m)$ is simplified to P_3. One must first guess these key bits to compute a dynamic variable. The right guess of key bits will lead to zero cube sums with high probability, otherwise the cube sums will be random.

3.3 Conditional Differential Cryptanalysis

Knellwolf, Meier and Naya-Plasencia [KMN10] applied the conditional differential characteristic to NFSR-based constructions and extended it to higher order differential attacks at ASIACRYPT 2010. The input of a synchronous stream cipher is an initial value (IV) and a key. Suppose that the keystream for many chosen IVs under the same secret key can be observed. By imposing specific conditions on certain bits of the IV, the attacker can control the propagation of a difference in the IV through the first few rounds of the initialization process. Taking IV pairs conforming to these conditions as input, the resulting keystream differences will present a bias. Additionally, conditions on the key define classes of weak keys.

3.4 Conditional Cube Attack

The conditional cube attack [HWX+17] was proposed by Huang *et al.* at EUROCRYPT 2017 to attack the KECCAK keyed mode. Inspired by the dynamic cube attack [DS11], which reduces the algebraic degree of output polynomials of cube variables by adding some bit conditions on the initial value (IV), they reduce the algebraic degree by adding key bit conditions. The techniques are similar to the message modification technique [WY05, WYY05] and conditional differential cryptanalysis [KMN10] which used bit conditions to control differential propagation.

Definition 1. ([HWX+17]) Cube variables which are not multiplied with each other in the first two rounds of KECCAK are called **conditional cube variables**. Cube variables that are not multiplied with each other in the first round and are not multiplied with any conditional cube variable in the second round are called **ordinary cube variables**.

Theorem 1. *([HWX+17]) For the $(n+2)$-round KECCAK sponge function $(n > 0)$, if there are p $(0 \leq p < 2^n+1)$ conditional cube variables $v_0, ..., v_{p-1}$, and $q = 2^{n+1}-2p+1$ ordinary cube variables, $u_0, ..., u_{q-1}$ (If $q = 0$, we set $p = 2^n + 1$), the term $v_0 v_1 ... v_{p-1} u_0 ... u_{q-1}$ will not appear in the output polynomials of the $(n + 2)$-round KECCAK sponge function.*

Actually, in the previous conditional cube attacks [HWX+17, LBDW17, SGSL18], they only use the special case of the above theorem when $p = 1$. We describe it as a corollary for clearness.

Corollary 1. *For the* $(n+2)$*-round* KECCAK *sponge function* $(n > 0)$*, if there is one conditional cube variable* v_0*, and* $q = 2^{n+1} - 1$ *ordinary cube variables* $u_0, ..., u_{q-1}$*, the term* $v_0 u_0 ... u_{q-1}$ *will not appear in the output polynomials of the* $(n+2)$*-round* KECCAK *sponge function.*

3.5 MILP Model of Conditional Cube Attack

Recently, the cryptographic community found that many classical cryptanalysis methods could be converted to mathematical optimization problems which aim to achieve the minimal or maximal value of an objective function under certain constraints. Mixed-integer Linear Programming (MILP) is the most widely studied technique to solve these optimization problems. One of the most successful applications of MILP is to search differential and linear trails. Mouha *et al.* [MWGP11] and Wu *et al.* [WW11] first applied the MILP method to count active S-boxes of word-based block ciphers. Then, at ASIACRYPT 2014, by deriving some linear inequalities through the H-Representation of the convex hull of all differential patterns of the S-box, Sun *et al.* [SHW+14] extended this technique to search differential and linear trails. Two other important applications are to search integral distinguishers [XZBL16] and impossible differentials [ST17, CJF+16].

At ASIACRYPT 2017, Li *et al.* [LBDW17] for the first time applied the MILP method to cube attacks on keyed KECCAK. Later, the MILP model was improved by [SGSL18], and also applied to the cube-attack-like method by Bi *et al.* [BDL+18] and Song *et al.* [SG18]. In the previous MILP model of the conditional cube attack, no cube variables multiply with each other in the first round, and the conditional cube variable of degree one does not multiply with any ordinary cube variables in the second round. To limit the diffusion of conditional cube variables, conditions are added in the first round. Actually, as described in Corollary 1, $(2^{n+1} - 1)$ ordinary cube variables are needed to perform a $(n+2)$-round attack. To obtain enough ordinary cube variables for the attack, the objective of Li *et al.*'s MILP model [LBDW17] at ASIACRYPT 2017 was to maximize the number of ordinary cube variables. To reduce the attack complexity further, Song *et al.* [SGSL18] proposed a new MILP model to minimize the number of conditions at ASIACRYPT 2018. We list the core relations among the MILP intermediate variables in Tables 4 and 5 in Appendix A.

4 New Conditional Cube Attack

In Huang *et al.*'s work [HWX+17] according to Corollary 1, the conditional cube attack is performed when all the cube variables do not multiply in the 1st round and one conditional cube variable does not multiply with others in the 2nd round under certain bit conditions. For $(n+2)$-round attacks, the 2^{n+1}-dimensional cube sum is zero when the bit conditions are satisfied.

Different from Huang *et al.*'s work [HWX+17], we remove the limitation that all the cube variables do not multiply in the 1st round. Actually, only two cube variables v_0, v_1 multiply in the 1st round, i.e. $v_0 v_1$ is the unique quadratic term in the output of the 1st round. Under certain bit conditions, the unique quadratic term $v_0 v_1$ does not multiply with any other cube variable in the 2nd round. In the output of the 2nd round, the bit conditions make any cubic term containing $v_0 v_1$ disappear. For $(n+2)$-round attacks, the cube sum over $(2^{n+1} + 1)$ dimensions is zero when the bit conditions are satisfied. To give a detailed description, we give the following definition first.

Definition 2. Suppose all the $(q+2)$ cube variables are $v_0, v_1, u_0, ...u_{q-1}$, and the constraints are as follows:

- In the output of the first round, v_0v_1 is the only quadratic term;
- In the second round, if the bit conditions are satisfied, v_0v_1 does not multiply with any of $u_0, ...u_{q-1}$, i.e. no cubic term occurs.
- In the second round, if the bit conditions are not satisfied, v_0v_1 multiplies with some of $u_0, ...u_{q-1}$, i.e. some cubic terms like $v_0v_1u_i$ $(i = 0, ..., q-1)$ occur.

Then v_0v_1 is called **kernel quadratic term**. The remaining cube variables except v_0 and v_1, i.e. $u_0, ...u_{q-1}$, are called **ordinary cube variables**.

Then we describe the new conditional cube attack using the **kernel quadratic term** in the following corollary.

Corollary 2. *For the $(n+2)$-round* KECCAK *sponge function $(n > 0)$, if there is one kernel quadratic term v_0v_1, and $q = 2^{n+1} - 1$ ordinary cube variables, $u_0, u_1, ..., u_{q-1}$, the term $v_0v_1u_0u_1...u_{q-1}$ will not appear in the output polynomials of the $(n+2)$-round* KECCAK *sponge function under certain bit conditions.*

Proof. Note that the degree of the KECCAK round function is two, so the upper bound of the degree of the output of $(n+2)$-round KECCAK is 2^{n+2}. However, we only selected $q+2$ cube variables, so the highest possible output degree of $(n+2)$ rounds is $q+2 = 2^{n+1}+1 \ll 2^{n+2}$.

According to Definition 2, if the bit conditions are satisfied, the degree of the output polynomials of the 2-round KECCAK-p permutation is two. So the output degree of n+2 rounds is no more than 2^{n+1}. So the term $v_0v_1u_0u_1...u_{q-1}$ with degree $q+2 = 2^{n+1}+1$ will not appear in the output polynomials of the $(n+2)$-round KECCAK sponge function under the bit conditions. □

Hence the distinguisher works as follows:

- under the right bit conditions, the degree of the output of $(n+2)$-round KECCAK is no more than 2^{n+1};
- under wrong bit conditions, the degree of the output of $(n+2)$-round KECCAK is $q+2 = 2^{n+1}+1$.

A key question is whether the output degree of n+2 rounds could reach $2^{n+1}+1$ under wrong bit conditions. Our experiments on 6-round KETJE SR v2 show that the output degree could always reach the highest possible degree, i.e. $q+2 = 2^{4+1}+1 = 33$. This is due to the very good diffusion of KECCAK round function. We assume that 7-round KECCAK has a similar property to 6-round KECCAK and the distinguisher also holds in the attack on 7-round KECCAK. In fact, all previous conditional cube attacks [HWX+17, LDW17, LBDW17, SGSL18] share similar assumptions.

The source code of searching the 33-dimensional cube satisfying the constraints in Definition 2 for KETJE SR v2, as well as the verification and the key-recovery attack on 6-round KETJE SR v1 and v2 are given in https://github.com/lizhengcn/Test_on_6round_KetjeSR, while the details of experiments are in Subsection 5.2.

4.1 New 6-2-2 Pattern

In the KECCAK submission document [BDPVA09], the authors introduce the CP-kernel as follows: if all columns in a state have even parity, θ is the identity. If $\bigoplus_{j=0}^{4} A[i][j][k] = 0$, θ is also the identity for variables in the column $(i, *, k)$. For example, if $A[i][0][k] =$

$v_1, A[i][1][k] = v_2, A[i][2][k] = v_3, A[i][3][k] = v_4, A[i][4][k] = v_1 \oplus v_2 \oplus v_3 \oplus v_4$, θ acts as the identity for variables in the column $(i, *, k)$ owing to $\bigoplus_{j=0}^{4} A[i][j][k] = 0$.

In the state $S_{1,\pi}$ of the 2nd round, if the conditional cube variables occupy fewer bit positions, we will have more chances to find the ordinary cube variables , which have to be not multiplied with the conditional cube variables after 2 rounds.

In Huang *et al.*'s work [HWX+17], as the left part of Figure 5 shows[1], one *conditional cube variable* v_0 is placed in two black bits of S_0, i.e. $S_0[2][0][0] = S_0[2][1][0] = v_0$, which is exactly set in the CP-kernel. After adding some conditions, the conditional cube variable v_0 maintains only 2 active bits in S_1 and diffuses to 22 bits after the linear part θ, ρ, π of the 2nd round as shown in state $S_{1,\pi}$. The diffusion pattern is denoted as 2-2-22. This pattern provides a nice restriction on the diffusion of the conditional cube variable v_0. Actually, the 2-2-22 patten is chosen by all the previous conditional cube attacks for Keccak keyed modes [HWX+17, LBDW17, SGSL18].

In our new conditional cube attack, we only care that the *kernel quadratic term* (i.e. v_0v_1) should not be multiplied with ordinary cube variables in the second round. Therefore, in order to get more degrees of freedom to find ordinary cube variables, we have to reduce the diffusion of v_0v_1. Hence, we introduce the so-called *6-2-2 pattern* shown in the right part of Figure 5. In the initial state S_0, v_0 occupies 4 black bits of S_0, i.e. $S_0[2][0][0] = S_0[2][1][0] = S_0[3][0][34] = S_0[3][1][34] = v_0$, which are also set in the CP-kernel, and v_1 occupies the other 2 grey bits, i.e. $S_0[0][1][60] = S_0[1][1][1] = v_1$.

In the χ operation of the 1st round, the *kernel quadratic term* v_0v_1 is generated, as shown in Figure 6. Before the χ operation, v_0 (initialized in the CP-kernel) only appears in the 4 black bits, while v_1 appears in 22 grey bits. Only 2 S-boxes highlighted by red rectangles are related to v_0 and v_1 simultaneously. According to the expression of the χ operation: $A[x][y] = A[x][y] \oplus ((\neg A[x+1][y]) \wedge A[x+2][y]$, the quadratic term v_0v_1 is just generated in two bits in black slashes after the χ operation.

The bit positions of v_0v_1 are also shown in S_1 (the output state of the 1st round) in the right part of Figure 5. Considering the quadratic term v_0v_1, the two bits in S_1 are in the CP-kernel, thus the θ operation in the 2nd round is the identity regarding v_0v_1. Then the second ρ and π just permute the positions of these two bits as shown in $S_{1,\pi}$ of Figure 5.

In Corollary 1 of Huang *et al.*'s attacks, to perform an attack on a $(n+2)$-round Keccak sponge function $(n > 0)$, finding $q = 2^{n+1} - 1$ other cube variables that do not multiply with v_0 even in the second round is the basis of the conditional cube attack. While the basis of the new conditional cube attack shown in Corollary 2 is quite similar, it is to find $q = 2^{n+1} - 1$ ordinary cube variables that are not multiplied with v_0v_1 even in the second round. Thus, through $S_{1,\pi}$ shown in both parts of Figure 5, it is obvious that the search for ordinary cube variables that do not multiply with v_0v_1 in the 6-2-2 pattern is much easier than that with v_0 in the 2-2-22 pattern. As a result, we achieve attacks on 7-round Keccak-MAC-512, i.e., one more round than the previous conditional cube attacks.

The method to find 6-2-2 patterns. As shown in Figure 5, v_0v_1 is in the CP-kernel in S_1. Denote the bit positions of v_0v_1 as (x, y_0, z), (x, y_1, z). According to the expression of χ, v_0v_1 in $S_1[x][y_0][z]$ is generated by multiplying v_0 in $S_{0,\pi}[x+1][y_0][z]$ and v_1 in $S_{0,\pi}[x+2][y_0][z]$, or v_0 in $S_{0,\pi}[x+2][y_0][z]$ and v_1 in $S_{0,\pi}[x+1][y_0][z]$. The same happens to v_0v_1 in $S_1[x][y_1][z]$. So there will be 4 cases to determine the bit positions for v_0 and v_1 in reverse. For example, in Figure 7, v_0v_1 appears in $S_1[1][2][z]$ by multiplication of v_0 in $S_{0,\pi}[2][2][z]$ and v_1 in $S_{0,\pi}[3][2][z]$, and similarly v_0v_1 appears in $S_1[1][4][z]$ by multiplication of v_1 in $S_{0,\pi}[2][4][z]$ and v_0 in $S_{0,\pi}[3][4][z]$.

Under one of the 4 cases, Table 3 describes the bit positions of v_0v_1, v_0 and v_1 inversely from S_1 to $S_{0,\theta}$, while the other cases are similar. In Table 3, in order to reduce the diffusion of v_0, we also use the CP-kernel property. Thus we add 2 bits containing v_0 (bold

[1]For the 1600-bit state, the bit $A[x][y][z]$ is located at coordinates $(5y + x, z)$ in Figure 5 and Figure 6.

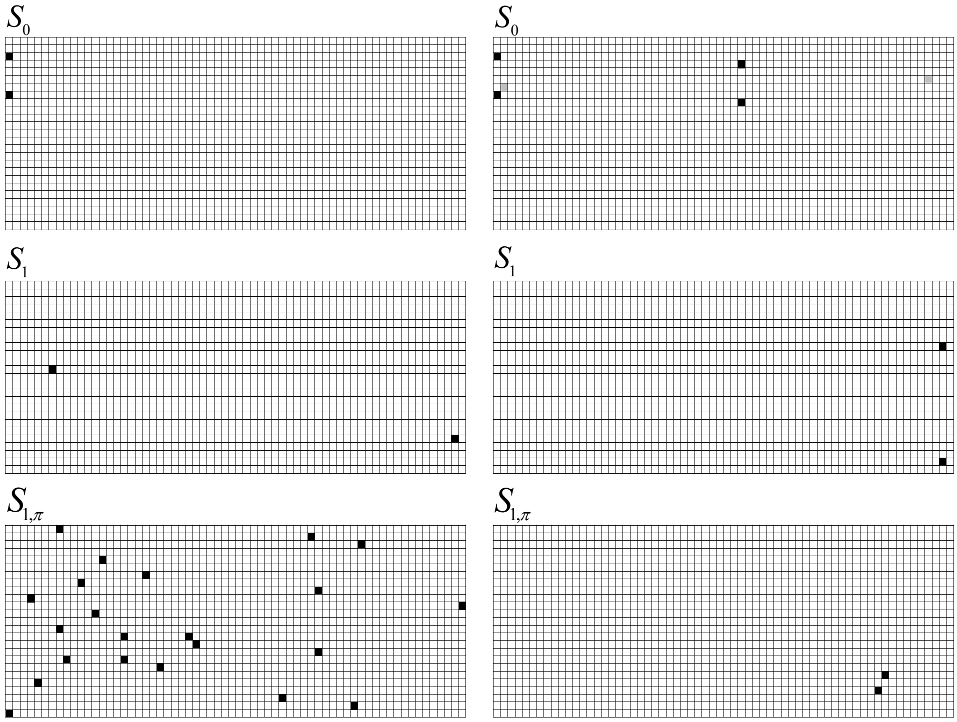


Figure 5: Diffusions of the conditional cube variable in 2-2-22 and 6-2-2 pattern in Keccak-MAC-512

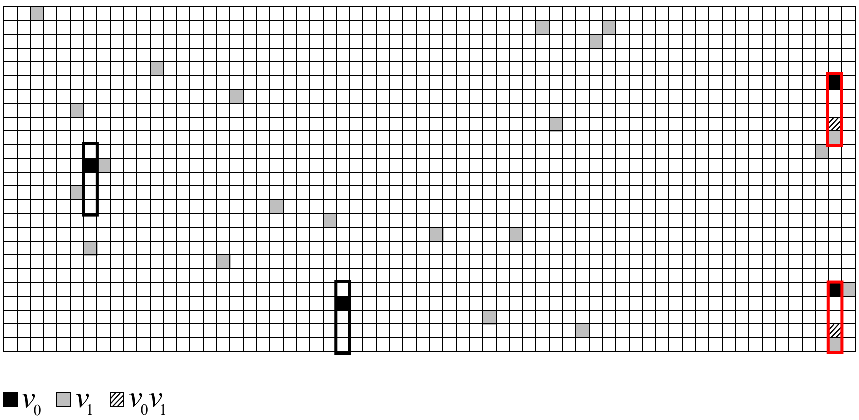


Figure 6: 6-2-2 pattern: generation of kernel quadratic terms in the first χ

ones in Table 3) in S_0, where $x_1, x_2 \neq x+1$. At last, all the 6 bits in S_0 should be selected in the free space for ordinary cube variables. Therefore, we can obtain the bit positions shown in Table 3, i.e. the 6-2-2 patterns.

4.2 MILP Model of the New Conditional Cube Attack

The 6-2-2 pattern has already determined the bit positions of v_0 and v_1 in the initial state, as well as the bit positions of the *kernel quadratic term* v_0v_1 in the output of the

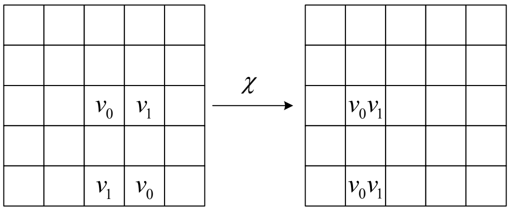


Figure 7: Slice $(*, *, z)$: from $S_{0,\pi}$ to S_1

Table 3: Related indexes of bits containing v_0, v_1 and v_0v_1

Index	v_0v_1	
S_1	(x, y_0, z) (x, y_1, z)	
Index	v_0	v_1
$S_{0,\pi}$	$(x+1, y_0, z)$ $(x+1, y_1, z)$	$(x+2, y_0, z)$ $(x+2, y_1, z)$
$S_{0,\rho}$	$(x+3y_0+1, x+1, z)$ $(x+3y_1+1, x+1, z)$	$(x+3y_0+2, x+2, z)$ $(x+3y_1+2, x+2, z)$
$S_{0,\theta}$	$(x+3y_0+1, x+1, z-\rho[x+3y_0+1, x+1])$ $(x+3y_1+1, x+1, z-\rho[x+3y_1+1, x+1])$	$(x+3y_0+2, x+2, z-\rho[x+3y_0+2, x+2])$ $(x+3y_1+2, x+2, z-\rho[x+3y_1+2, x+2])$
S_0	$(x+3y_0+1, x+1, z-\rho[x+3y_0+1, x+1])$ $\boldsymbol{(x+3y_0+1, x_1, z-\rho[x+3y_0+1, x+1])}$ $(x+3y_1+1, x+1, z-\rho[x+3y_1+1, x+1])$ $\boldsymbol{(x+3y_1+1, x_2, z-\rho[x+3y_1+1, x+1])}$	$(x+3y_0+2, x+2, z-\rho[x+3y_0+2, x+2])$ $(x+3y_1+2, x+2, z-\rho[x+3y_1+2, x+2])$

first round. To perform an attack on a $(n+2)$-round Keccak sponge function $(n > 0)$, the remaining $2^{n+1}-1$ ordinary cube variables and related conditions are found by the following MILP model.

The notations of the corresponding state in the model are similar to the previous ones used in [LBDW17, SGSL18]. We list the involved symbols here: Boolean variable $a[x][y][z] = 1$ means initial state bit $S_0[x][y][z] = A[x][y][z]$ contains at least one ordinary cube variable; Boolean variable $b[x][y][z] = 1$ means that of state bit $S_{0,\pi}[x][y][z]$, and Boolean variable $c[x][y][z]$ means that of state bit $S_1[x][y][z]$. Another two Boolean variables $v[x][y][z]$ and $h[x][y][z]$ are used to determine bit conditions:

- If $v[x][y][z] = 1$, bit condition "$S_{0,\pi}[x][y][z] = h[x][y][z]$" is added to the attack;
- If $v[x][y][z] = 0$, there are no bit conditions on $S_{0,\pi}[x][y][z]$.

Note that the sum of $v[x][y][z]$ means the number of bit conditions needed in our attack. If the bit condition "$S_{0,\pi}[x][y][z] = h[x][y][z]$" is related to a key bit, we need to guess it to assign a correct bit condition. In order to reduce the time complexity, we need to minimize the number of bit conditions that are related to key bits. Thus, the aim of the MILP model is to minimize the objective function:

$$\sum_{(S_{0,\pi}[x][y][z]/k) \neq 0} v[x][y][z],$$

where k represents any key bit in K, and $(S_{0,\pi}[x][y][z]/k) \neq 0$ means at least one key bit appears in $S_{0,\pi}[x][y][z]$. Thus the sum is exactly the number of key bits to be guessed when assigning correct bit conditions.

Modeling the First Round

When constraining the properties of the first θ operation, we use similar constraints as in [SGSL18]. $F[x][z]$ and $G[x][z]$ represent properties of column $(x, *, z)$. $G[x][z] = 1$ means that the sum of 5 bits in column $(x, *, z)$ of the initial state A is nonzero, while $G[x][z] = 0$

means that the sum is zero. $F[x][z] = 1$ represents the case that some variables exist in column $(x, *, z)$ but the sum of 5 bits in the column is zero, while $F[x][z] = 0$ represents all the other cases. The corresponding system of inequalities are listed in Equation 2 in Appendix A.

However, we illustrate the following constraints different from the previous works. In our model, v_0 and v_1 are supposed to multiply with each other in the 1st round. In addition, constraints should be added to prevent v_0 or v_1 from multiplying by ordinary cube variables. Take the 6-2-2 pattern in Table 3 as an example: in S_0, 4 bits contain v_0, 2 bits contain v_1. After the θ, ρ and π operations, v_0 appears in 4 bits of $S_{0,\pi}$, and v_1 occupies at most 22 bits of $S_{0,\pi}$. Let set $\mathbb{T}$ contain the above bit positions in $S_{0,\pi}$, $|\mathbb{T}| \le 26$. According to the χ operation, only neighbouring bits in the same S-box will be multiplied together. Thus, for any $(x, y, z) \in \mathbb{T}$, add constraints $b[x-1][y][z] = b[x+1][y][z] = 0$.

For the other bits in $S_{0,\pi}$ instead of the above cases, the constraints should be added as in the previous MILP model. We list the corresponding values and constraints in Table 4 and Equation 3 in Appendix A.

Modeling the Second Round

In the input state of the χ operation in the second round (i.e. $S_{1,\pi}$), 2 bits contain the **kernel quadratic term** v_0v_1. According to the expression of the χ operation, only two adjacent bits multiply with each other. Thus some ordinary cube variables in the 4 bits next to the 2 bits containing v_0v_1 in $S_{1,\pi}$ should satisfy the following rules:

1. If the bit conditions are satisfied, no ordinary cube variables out of $u_0, ...u_{q-1}$ multiply with v_0v_1 in the second round.
2. If the bit conditions are not satisfied, some ordinary cube variables out of $u_0, ...u_{q-1}$ multiply with v_0v_1 in the second round.

Even though v_0 or v_1 multiply with the terms containing v_0v_1, the degree of corresponding terms containing v_0v_1 remains 2. So we omit the distribution of single v_0 and v_1 in the second round.

If a bit $S_0[x][y][z]$ contains any ordinary cube variables, $A[x][y][z] = 1$, otherwise $A[x][y][z] = 0$. However, for a set of more than one bit, a dummy variable is introduced. If any bit in the set contains any ordinary cube variables, the corresponding dummy variable equals 1, otherwise it equals 0. The 4 bit positions in $S_{1,\pi}$ next to the 2 bits containing v_0v_1 are derived from $S_{1,\theta}$ through bit shift. The corresponding 4 bit positions in $S_{1,\theta}$ are denoted as $S_{1,\theta}[x_i][y_i][z_i], i = 0, 1, 2, 3$. We introduce dummy variables $e[i]$, $i = 0, 1, 2, 3$, to avoid unexpected propagations for the 4 bits, which is similar to [SGSL18]. According to the expression of the θ operation,

$$S_{1,\theta}[x_i][y_i][z_i] = S_1[x_i][y_i][z_i] + \sum_{y=0,1,2,3} S_1[x_i+4][y][z_i] + \sum_{y=0,1,2,3} S_1[x_i+1][y][z_i-1]. \quad (1)$$

Boolean variable $c[x][y][z] = 1$ indicates state bit $S_1[x][y][z]$ contains at least one ordinary cube variable. In Equation 1, there are 11 bits of S_1, hence there will be 11 Boolean variables $c[x][y][z]$. If at least one Boolean variable $c[x][y][z]$ is 1, then $e[i] = 1$. In this case, one linear equation (the XOR of the 11 bits equals a constant, i.e. 0 or 1) should be satisfied, thus it consumes one bit degree of freedom. Therefore, when computing the number of cube variables, subtraction of $\sum_{i=0}^{3} e[i]$ is needed. If all of the 11-bit c equal 0, $e[i] = 0$.

The details of restrictions and values are listed in Equation 4 and Table 5 of Appendix A. Similar to [SGSL18], to obtain enough ordinary cube variables, the following constraint is also added in our model:

$$\sum_{x,y,z} A[x][y][z] - \sum_{x,z} F[x][z] - \sum_{i=0,1,2,3} e[i] = 2^{n+1} - 1.$$

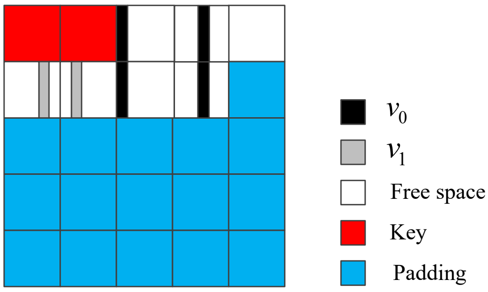


Figure 8: The initial state of KECCAK-MAC-512

5 Applications on Round-Reduced Keccak Keyed Modes

5.1 Attack on 7-round Keccak-MAC-512

For KECCAK-MAC-512, a 7-round attack can be performed with 65 cube variables. In the whole 1600-bit state, the rate occupies 576 bits, and the capacity 1024 bits. Suppose v_0v_1 is the *kernel quadratic term*. As Figure 8 shows, the 128-bit key K is located at the first 2 red lanes, and v_0 is set in the CP-kernel as $S_0[2][0][0] = S_0[2][1][0] = S_0[3][0][34] = S_0[3][1][34] = v_0$ in 4 black bits, while v_1 is located at 2 grey bits, i.e. $S_0[0][1][60] = S_0[1][1][1] = v_1$. The white part represents free space for ordinary cube variables, i.e. nonce bits which can be selected as ordinary cube variables, while the padding is in blue. According to the new conditional cube attack illustrated in Section 4, we search for the minimal number of key bit conditions and ordinary cube variables satisfying the corresponding rules.

Suppose the state S_0 is initialized by symbols, where the key bits are $k_0, k_1...k_{127}$, the following 448 bits are $n_0, n_1...n_{447}$, and each padding bit is 0 or 1. Through symbolic operation, each bit of $S_{0,\pi}$ is a polynomial. Take $S_{0,\pi}[x][y][z]$ as an example. Considering its coefficient of k_i for $i = 0, 1...127$, if a certain coefficient is nonzero, $S_{0,\pi}[x][y][z]$ is related to some key bits. We list the lanes in $S_{0,\pi}$ whose bits are related to some key bits in a set $\mathbb{S}$:

$$\begin{aligned}\mathbb{S} = \{&(0,0,*),(1,0,*),(2,0,*),(4,0,*),(1,1,*),\\ &(2,1,*),(3,1,*),(4,1,*),(0,2,*),(1,2,*),\\ &(3,2,*),(4,2,*),(0,3,*),(1,3,*),(2,3,*),\\ &(3,3,*),(0,4,*),(2,4,*),(3,4,*),(4,4,*)\}.\end{aligned}$$

The objective function to be minimized is

$$\sum_{(x,y)\in\mathbb{S}} v[x][y][z],$$

As at least one key bit condition is needed, the following constraint should be added:

$$\sum_{(x,y)\in\mathbb{S}} v[x][y][z] \geq 1.$$

The ordinary cube variables are denoted as $u_0, u_1...u_{62}$, and the number of ordinary cube variables is 63. The following equation is the constraint for enough ordinary cube variables:

$$\sum_{x,y,z} A[x][y][z] - \sum_{x,z} F[x][z] - \sum_{i=0,1,2,3} e[i] = 63.$$

The other constraints illustrated in Section 4 should be added. With the help of Gurobi [Gur], the objective function is optimized under all the above constraints, i.e.

with 63 ordinary cube variables, the minimal number of key bit conditions is 1. Actually, v_0 and v_1 are determined by the 6-2-2 pattern, and $u_0, u_1 ... u_{62}$ are obtained by solving the MILP model. Then, 65 cube variables are enough to perform the 7-round attack on Keccak-MAC-512. Both the cube variables and conditions are listed in Table 6 in Appendix B.

In the 7-round attack on Keccak-MAC-512, $2^6 + 1 = 65$ cube variables are denoted by $v_0, v_1, u_0, u_1 ... u_{62}$. Based on Corollary 2, v_0 and v_1 are elements of the kernel quadratic term fixed in the beginning and $u_0, u_1 ... u_{62}$ are ordinary cube variables found by the MILP search strategy. We summarize the requirements as follows:

(1) Only v_0 and v_1 multiply with each other in the first round;

(2) Under some conditions on key and nonce, $v_0 v_1$ does not multiply with any of $u_0, u_1 ... u_{62}$ in the second round;

(3) Under the conditions of the nonce in (2), if the key conditions are not satisfied, $v_0 v_1$ multiplies with some u_i $(i = 0, 1, ..., 62)$ in the second round.

According to the requirements above, the *kernel quadratic term* $v_0 v_1$ is the unique quadratic term in the output of the first round (i.e. S_1). So in the output of the second round, the available degree is at most 3 and the corresponding term should contain $v_0 v_1$ as a factor. As illustrated in the above requirements (2) and (3), under the conditions of the nonce, all cubic terms disappear with the key conditions satisfied, while some cubic terms like $v_0 v_1 u_i$ $(i = 0, 1, ..., 62)$ appear without the key conditions. As the algebraic degree of the round function in Keccak-p permutation is 2, the highest degree of terms in the output of the 7-round Keccak-MAC-512 (i.e. S_7) is 2^6 with the bit conditions satisfied, while the highest degree of S_7 is $2^6 + 1 = 65$ in the case that the key bit conditions are not satisfied. Therefore, if the 65-dimensional cube sums of 7-round output bits are 0, we conjecture that the key guess is correct with very high probability; if the term $v_0 v_1 u_0 ... u_{62}$ appears, the key guess is wrong.

While all the nonce bits are constant, all the bit conditions are satisfied if and only if all the key bits are guessed correctly. Thus, the key guess is conjectured to be correct when the cube sum over the 128-bit tag is zero, while the parameter is set as in Table 6 in Appendix B.

We analyze the time and data complexity of the 7-round attack on Keccak-MAC-512: with the parameter sets in Table 6, one guessed key bit k_{33} can be recovered. The time complexity of one recovery is $2^1 \times 2^{65}$. Due to the properties of the permutation, there is a complete symmetry in direction of the z-axis. Thus we can obtain corresponding parameter sets with any rotation index i $(0 \leq i < 64)$ in the z-axis. Therefore, the guessed key bits rotated by i bits, i.e., k_{i+33} can be recovered. Through simple count, for $0 \leq i < 64$, 64 independent key bits (i.e. the whole first lane) out of 128 key bits can be recovered, 64 iterations consume $64 \times 2^1 \times 2^{65}$ and the remaining 64 key bits are left to exhaustive search consuming 2^{64}. Combining the two parts, the procedure consumes $64 \times 2^1 \times 2^{65} + 2^{64} = 2^{72}$ computations of 7-round Keccak-MAC-512, and each one corresponds to a unique value of the input. After the procedure above, all the 128 bits in K can be recovered. Therefore, both time and data complexity of the attack are 2^{72}.

5.2 Attack on Round-Reduced Ketje Sr

For Ketje Sr with 400-bit state, the length of the key K is 128 bits, while the shortest padding occupies 18 bits. For the state size of only 400 rather than 1600 bits, we are able to control the diffusion of v_1 by slightly tweaking the 6-2-2 pattern. So 4 bits containing v_0 are shown in black and 3 bits containing v_1 are shown in grey in Figure 9, Figure 10.

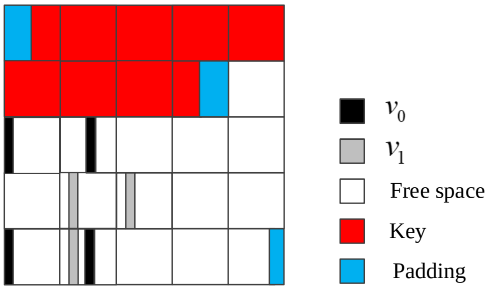


Figure 9: The initial state of KETJE SR v1

According to the new conditional cube attack illustrated in Section 4, similar to the attacks in Subsection 5.1, we search for the minimal number of key bit conditions and ordinary cube variables satisfying the corresponding rules. As each bit in $S_{0,\pi}[x][y][z]$ is related to some key bits, the objective function to be minimized is

$$\sum_{x,y\in\{0,1\ldots4\},z\in\{0,1\ldots63\}} v[x][y][z],$$

As at least one key bit condition is needed, the following constraint should be added:

$$\sum_{x,y\in\{0,1\ldots4\},z\in\{0,1\ldots63\}} v[x][y][z] \geq 1.$$

In the 6-round attack, the number of ordinary cube variables is 31, thus we add the following constraint:

$$\sum_{x,y,z} A[x][y][z] - \sum_{x,z} F[x][z] - \sum_{i=0,1,2,3} e[i] = 31.$$

In the 7-round attack, the number of ordinary cube variables is 63, thus we add the following constraint:

$$\sum_{x,y,z} A[x][y][z] - \sum_{x,z} F[x][z] - \sum_{i=0,1,2,3} e[i] = 63.$$

The other constraints illustrated in Section 4 should be added. With the help of Gurobi [Gur], the objective function is optimized under all the above constraints.

5.2.1 Attack on Round-Reduced Ketje Sr v1

Suppose v_0v_1 is the *kernel quadratic term*. As Figure 9 shows, the 128-bit key is located at the red parts, while the padding part is shown in blue. and v_0 is set in CP-kernel as $S_0[0][2][0] = S_0[0][4][0] = S_0[1][2][9] = S_0[1][4][9] = v_0$ in black, and v_1 is located at 3 grey bits, i.e. $S_0[1][3][6] = S_0[1][4][6] = S_0[2][3][4] = v_1$. The white bits represent free space to be selected as ordinary cube variables.

Attack on 6-round Ketje Sr v1

With 31 ordinary cube variables, the minimal number of key bit conditions is 1, thus a 6-round attack on KETJE SR v1 can be performed. Both the cube variables and conditions are listed in Table 7 in Appendix B.

We analyze the time and data complexity of the attack: with the parameter sets in Table 7 in Appendix B, the guessed key bit $k_{39} + k_{70} + k_{119}$ can be recovered. The time complexity of one recovery is $2^1 \times 2^{33}$. According to the property of the permutation, it is totally symmetric in the z-axis. Thus we can obtain the corresponding parameter sets with any i-bit rotation ($0 \leq i < 64$) in the z-axis. Therefore, the guessed key bits rotated by i bits, i.e. $k_{i+39} + k_{i+70} + k_{i+119}$ can be recovered. Actually, as different choices of the 7-2-2 pattern with different v_0 and v_1 are possible, we have different choices for the predetermined parameters in the MILP model. Meanwhile, there are plenty of solutions of the MILP model with one set of predetermined parameters. Therefore, other sets of parameters related to different key bits are easily obtained, and we can recover 96 key bits. 96 iterations consume $96 \times 2^1 \times 2^{33}$ and the remaining 32 key bits are left to exhaustive search, consuming 2^{32}. Combining the two parts, the procedure consumes $2^{40.6}$ computations of 6-round Ketje Sr v1, and each one corresponds to a unique value of the input. After the procedure above, all the 128 bits in K can be recovered. Therefore, both time and data complexity of the attack are $2^{40.6}$.

We give an example here for intuition, in which the key is generated randomly and all the controllable nonce bits are set to zero.

128-bit key K:

`1010000011010110011101001101110001110010000111011101110010110110`

`1111110010011101001011000101010100010111101000111100101100000101`

According to Table 7 in Appendix B, with bit conditions satisfied, 33 cube variables takes 2^{33} different values. Based on the guessed value of the key bit $k_{39} + k_{70} + k_{119}$, we observed the following cube sums:

Guessed value	Cube sums
0	0xaa1b, 0x46d
1	0x0, 0x0

If the guessed value equals 0, the cube sums are not zero; if the guessed value equals 1, the cube sums are zero. Therefore, we conjecture the value of the key bit $k_{39} + k_{70} + k_{119}$ is 1, and it is correct. Actually, all the 1000 experiments performed the key recovery attacks of 6-round Ketje Sr v1 correctly. The program is run in Visual Studio 2010 with x64 platform Release. The time is about 6 hours for recovery of one key bit using one CPU core (Intel i7 3.6GHz), and parallelism can reduce time. The test code is provided in `https://github.com/lizhengcn/Test_on_6round_KetjeSR`.

Attack on 7-round Ketje Sr v1

With 63 ordinary cube variables, the minimal number of key bit conditions is 7, thus the attack on 7-round Ketje Sr v1 can be performed. Both the cube variables and conditions are listed in Table 8 in Appendix B. We analyze the time and data complexity of the attack: with the parameter sets in Table 8, the 7 guessed key bits $k_8 + k_{57} + k_{88}$, $k_{13} + k_{44} + k_{93} + k_{124}$, $k_{39} + k_{70} + k_{119}$, $k_{26} + k_{75} + k_{91} + k_{106}$, $k_{21} + k_{52} + k_{101}$, $k_{15} + k_{46} + k_{95} + k_{111} + k_{126}$, $k_9 + k_{40} + k_{89} + k_{120}$ can be recovered. The time complexity of one recovery is $2^7 \times 2^{65}$. According to the property of permutation, it is totally symmetric in the z-axis. Thus we can obtain the corresponding parameter sets with any i-bit rotation ($0 \leq i < 64$) in the z-axis. Therefore, the guessed key bits rotated by i bits can be recovered. Combining procedures of key recovery by related parameters and exhaustive search, the procedure consumes 2^{75} computations of 7-round Ketje Sr v1, and each one corresponds to a unique value of the input. After the procedure above, all the 128 bits in K can be recovered. Therefore, both time and data complexity of the attack are 2^{75}.

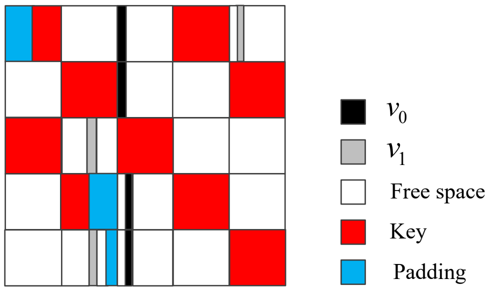


Figure 10: The state after π^{-1} of Ketje Sr v2

5.2.2 Attack on Round-Reduced Ketje Sr v2

Suppose v_0v_1 is the *kernel quadratic term.* As Figure 10 shows, the 128-bit key is located at the red parts, while the padding part is shown in blue. And v_0 is set in the CP-kernel as $S_0[2][0][0] = S_0[2][1][0] = S_0[2][3][1] = S_0[2][4][1] = v_0$ in black, and v_1 is located at 3 grey bits, i.e. $S_0[1][2][12] = S_0[1][4][12] = S_0[4][0][1] = v_1$. The white bits represent nonce bits which can be selected as ordinary cube variables.

Attack on 6-round Ketje Sr v2

With 31 ordinary cube variables, the minimal number of key bit conditions is 1, thus a 6-round attack on Ketje Sr v2 can be performed. Both the cube variables and conditions are listed in Table 9 in Appendix B. We analyze the time and data complexity of the attack: with the parameter sets in Table 9 in Appendix B, the guessed key bit $k_{34} + k_{65} + k_{82} + k_{97}$ can be recovered. The time complexity of one recovery is $2^1 \times 2^{33}$. According to the property of permutation, it is totally symmetric in the z-axis. Thus we can obtain corresponding parameter sets with any i-bit rotation ($0 \leq i < 64$) in the z-axis. Therefore, the guessed key bits rotated by i bits i.e. $k_{i+34} + k_{i+65} + k_{i+82} + k_{i+97}$ can be recovered. By a computation similar to Ketje Sr v1, both time and data complexity of the attack are $2^{40.6}$.

We give an example here for intuition, in which the key is generated randomly and all the controllable nonce bits are set to zero.

128-bit key K:

`0111111100001001011011111010010011001100011011110111111111001000`
`0101010010011101110010110000110010110101110111011000111101000011`

According to Table 9 in Appendix B, with bit conditions satisfied, 33 cube variables takes 2^{33} different values. Based on the guessed value of the key bit $k_{34} + k_{65} + k_{82} + k_{97}$, we observed the following cube sums:

Guessed value	Cube sums
0	0x3182, 0xbd7
1	0x0, 0x0

If the guessed value equals 0, the cube sums are not zero; if the guessed value equals 1, the cube sums are zero. Therefore, we conjecture the value of the key bit $k_{34} + k_{65} + k_{82} + k_{97}$ is 1, and it is correct. Actually, all the 1000 experiments performed the key recovery attacks of 6-round Ketje Sr v2 correctly. The program is run in Visual Studio 2010 with x64 platform Release. The time is about 6 hours for recovery of one key bit using one

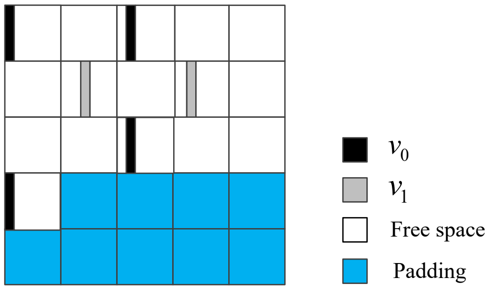


Figure 11: The state of KMAC256

CPU core (Intel i7 3.6GHz), and parallelism can reduce time. The test code is given in https://github.com/lizhengcn/Test_on_6round_KetjeSR.

Attack on 7-round Ketje Sr v2

With 63 ordinary cube variables, the minimal number of key bit conditions is 10, thus a 7-round attack on Ketje Sr v2 can be performed. Both the cube variables and conditions are listed in Table 10 in Appendix B. We analyze the time and data complexity of the attack: with the parameter sets in Table 10, the 10 guessed key bits $k_{28}+k_{109}$, $k_{27}+k_{108}$, $k_{39}+k_{104}$, $k_2+k_{33}+k_{114}$, $k_{39}+k_{70}+k_{87}+k_{102}$, $k_{25}+k_{56}+k_{88}$, $k_1+k_{32}+k_{113}$, $k_{26}+k_{107}$, $k_{28}+k_{59}+k_{91}$, $k_5+k_{36}+k_{117}$ can be recovered. The time complexity of one recovery is $2^{10}\times 2^{65}$. Therefore, the guessed key bits rotated by i bits can be recovered. Through similar computation, both time and data complexity of the attack are 2^{77}.

5.3 Attack on 9-round KMAC256

For KMAC256, a 9-round attack can be performed with 129 cube variables. The state of KMAC256 is 1600-bit, and the key size is 256-bit. The output length can be more than 320-bit, and the first 5 lanes of S_9 can be inverted through the χ operation of the 9th round, i.e. the first 5 lanes of $S_{8,\pi}$ can be obtained. Due to the linearity of θ, ρ, π, $S_{8,\pi}$ and S_8 share the same algebraic degree. Therefore, 129 cube variables can achieve 9-round attacks on KMAC256. The state just before the message is involved in the state to place cube variables. The whole state is secret while the length of the free nonce involved is 1048 bits. Suppose v_0v_1 is the *kernel quadratic term*. As Figure 11 shows, v_0 is set in the CP-kernel as $S_0[0][0][0]=S_0[0][3][0]=S_0[2][0][2]=S_0[2][2][2]=v_0$ in black, and v_1 is located at 2 grey bits, i.e. $S_0[1][1][20]=S_0[3][1][9]=v_1$, while the padding is in blue. The white bits provide free space for ordinary cube variables. According to the new conditional cube attack illustrated in Section 4, we search for the minimal number of key bit conditions and ordinary cube variables satisfying the corresponding rules. As each bit in $S_{0,\pi}[x][y][z]$ is related to some key bit, the objective function to be minimized is

$$\sum_{x,y\in\{0,1\ldots 4\},z\in\{0,1\ldots 63\}} v[x][y][z].$$

As at least one key bit condition is needed, the following constraint should be added:

$$\sum_{x,y\in\{0,1\ldots 4\},z\in\{0,1\ldots 63\}} v[x][y][z] \geq 1.$$

The ordinary cube variables are denoted as $u_0, u_1, \ldots, u_{126}$, and the number of ordinary cube variables is 127. The following equation is the constraint for enough ordinary cube variables:

$$\sum_{x,y,z} A[x][y][z] - \sum_{x,z} F[x][z] - \sum_{i=0,1,2,3} e[i] = 127.$$

The other constraints illustrated in Section 4 should be added. With the help of Gurobi [Gur], the objective function is optimized under all the above constraints, and the minimal number of key bit conditions is 1. Actually, v_0 and v_1 are selected by the 6-2-2 pattern, and the MILP model return the variables $u_0, u_1, ..., u_{126}$. Therefore, the above 129 cube variables are sufficient to perform the 9-round attack on `KMAC256`. Both the cube variables and conditions are listed in Table 11 in Appendix B. According to Corollary 2, $v_0 v_1 u_0 ... u_{126}$ appears without the key conditions; If the key bit condition is satisfied, term $v_0 v_1 u_0 ... u_{126}$ does not appear. Thus, with parameter sets as in Table 11 in Appendix B, if the cube sum over the known bits of $S_{8,\pi}$ is zero, we conjecture that the key bit condition is satisfied, i.e. the key guess is conjectured to be correct.

We analyze the time and data complexity of the 9-round attack on `KMAC256`: with the parameter sets in Table 11, if the key bit equals 0 (i.e. $k_{14} + k_{207} + k_{334} + k_{527} + k_{654} + k_{847} + k_{911} + k_{974} + k_{1167} + k_{1294} + k_{1487} + 1 = 0$), the cube sums are zero; if the key bit equals 1 (i.e. $k_{14} + k_{207} + k_{334} + k_{527} + k_{654} + k_{847} + k_{911} + k_{974} + k_{1167} + k_{1294} + k_{1487} + 1 = 1$), the cube sums are nonzero. So the key bit $k_{14} + k_{207} + k_{334} + k_{527} + k_{654} + k_{847} + k_{911} + k_{974} + k_{1167} + k_{1294} + k_{1487}$ can be recovered. The time complexity of one recovery is 2^{129}. According to the property of Keccak-p permutations, it is totally symmetric in the z-axis. Thus we can obtain corresponding parameter sets with any i-bit rotation ($0 \le i < 64$) in the z-axis. Therefore, we can recover the other key bits by a i-bit rotation. Through simple count, for $0 \le i < 64$, 64 independent key bits can be recovered, and 64 iterations consume 64×2^{129}. We can obtain different sets of parameters similarly, it is owing to the many choices of 6-2-2 pattern with different v_0, v_1 and numerous solutions by the MILP model. We omit them here, and totally 1470 iterations consume 1470×2^{129} to recover 1470 bits of the intermediate state. Then the remaining 130 state bits are left to exhaustive search consuming 2^{130}. Combining the two parts, the procedure consumes $1470 \times 2^{129} + 2^{130} = 2^{139}$ computations of 9 rounds of `KMAC256`, and each one corresponds to a unique value of the input. After the procedure above, all 1600 bits of the internal state can be recovered, so we can recover the initial state containing 256 key bits. Therefore, both time and data complexity of the attack are 2^{139}.

6 Conclusion

In this paper, we introduce a new conditional cube attack on Keccak keyed modes. In our new attack, we let quadratic terms occur in the first round, which is different from previous conditional cube attacks. In the second round, we add bit conditions so that the quadratic terms do not multiply with other cube variables. Thus, in the first two rounds, there are no cubic terms. Furthermore, we introduce the *kernel quadratic term* and construct a *6-2-2 pattern*, which reduces the diffusion of quadratic terms significantly. The ideas above make the operation θ in the 2nd round an identity transformation for the distribution of the kernel quadratic term, while the θ operation is the significant part providing diffusion. So more degrees of freedom are available to find enough ordinary cube variables, which is the key part to even validate the conditional cube attack on versions with few degrees of freedom. Furthermore, fewer bit conditions are used to make the cubic terms disappear. Owing to the above techniques, we can perform key-recovery attacks. The MILP method is also used in the search of ordinary cube variables.

Based on the above, we can attack 7-round Keccak-MAC-512, 7-round Ketje Sr v2 and 9-round `KMAC256` with time complexity 2^{72}, 2^{77}, and 2^{139}, respectively, which are faster than the best previous attack by a factor of 2^{39}, 2^{22} and 2^{8}. Focusing on the conditional cube attack, we have improved the attack on both round-reduced Keccak-MAC-512 and Ketje Sr v2 by one round.

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A Supplementary of Variables in MILP model: Values and Constraints for Some Cases

In this section, we list some values and corresponding constraints used in the MILP model here for a supplementary, as similar constraints are also used in [SGSL18], while they may be used for different scenes there. Equation 2 illustrates the constraints for ordinary cube variables (i.e. cube variables except v_0 and v_1) in θ operation of the first round.

$$\begin{cases} -F[x][z] - G[x][z] \geq \text{-1} \\ -A[x][0][z] + F[x][z] + G[x][z] \geq 0 \\ -A[x][1][z] + F[x][z] + G[x][z] \geq 0 \\ -A[x][2][z] + F[x][z] + G[x][z] \geq 0 \\ -A[x][3][z] + F[x][z] + G[x][z] \geq 0 \\ -A[x][4][z] + F[x][z] + G[x][z] \geq 0 \\ A[x][0][z] + A[x][1][z] + A[x][2][z] + A[x][3][z] + A[x][4][z] - 2F[x][z] - G[x][z] \geq 0 \end{cases} \tag{2}$$

We list the values of model variables for ordinary χ operation of the first round in Table 4, and corresponding constraints are listed in Equation 3.

Table 4: Summary of variables for ordinary χ operation in the 1st round

$B[x]$	$B[x+1]$	$B[x+2]$	$V[x+1]$	$V[x+2]$	$H[x+1]$	$H[x+2]$	$C[x]$
0	0	0	*	*	*	*	0
1	0	0	*	*	*	*	1
0	0	1	0	0	*	*	1
0	0	1	1	0	1	*	0
0	0	1	1	0	0	*	1
0	1	0	0	0	*	*	1
0	1	0	0	1	*	0	0
0	1	0	0	1	*	1	1
1	0	1	0	0	*	*	1
1	0	1	1	0	*	*	1

$$
\begin{cases}
-B[x]-B[x+1] \geq \text{-1} \\
-B[x]+C[x] \geq 0 \\
-B[x+2]-V[x+2] \geq \text{-1} \\
-B[x+1]-V[x+1] \geq \text{-1} \\
-B[x]-B[x+1]-H[x+2]+C[x] \geq \text{-1} \\
B[x]-V[x+1]-H[x+1]-C[x] \geq \text{-2} \\
B[x]-V[x+2]+H[x+2]-C[x] \geq \text{-1} \\
B[x]+B[x+1]+B[x+2]-C[x] \geq 0 \\
-B[x+1]-B[x+2]+V[x+1]+V[x+2]+C[x] \geq 0 \\
-B[x+1]-B[x+2]+V[x+2]+H[x+1]+C[x] \geq 0
\end{cases}
\tag{3}
$$

As we bring in dummy variables $e[i], i = 0, 1, 2, 3$ to record and eliminate some uncertain propagation of cube variables, the possible values of corresponding model variables are listed in Table 5, and corresponding constraints are listed are in Equation 4.

Table 5: Summary of dummy variables in the 2nd round

$e[i]$	$B[x][y][z]$	$B[x+1][y][z]$	$B[x+2][y][z]$	$V[x+1][y][z]$	$V[x+2][y][z]$
0	*	*	*	*	*
1	0	0	0	*	*
1	1	0	0	*	*
1	1	0	1	1	0
1	0	0	1	1	0
1	0	1	0	0	1

$$
\begin{cases}
-e[i]-B[x+1][y][z]-B[x+2][y][z] \geq \text{-2} \\
-e[i]-B[x+1][y][z]+V[x+2][y][z] \geq \text{-1} \\
-e[i]-B[x+2][y][z]+V[x+1][y][z] \geq \text{-1} \\
-e[i]-B[x+1][y][z]-V[x+1][y][z] \geq \text{-2} \\
-e[i]-B[x+2][y][z]-V[x+2][y][z] \geq \text{-2} \\
-e[i]-B[x][y][z]-B[x+1][y][z] \geq \text{-2}
\end{cases}
\tag{4}
$$

B Parameter Sets for Attacks on Round-Reduced Keccak-MAC-512, Ketje Sr v1&v2 and KMAC256

In this section, we list the parameter sets for attacks on round-reduced Keccak-MAC-512, Ketje sr v1&v2 and KMAC256.

Table 6: Parameter sets for attack on 7-round Keccak-MAC-512

kernel quadratic term
$A[2][0][0]{=}A[2][1][0]{=}A[3][0][34]{=}A[3][1][34]{=}v_0, A[0][1][60]{=}A[1][1][1]{=}v_1$
Bit Condition
$A[0][1][33]{=}k_{33}+A[1][1][33], A[2][0][52]{=}A[4][0][51]+A[2][1][52], A[4][0][0]{=}0$
Ordinary Cube Variables
$A[0][1][44]{=}u_0, A[1][1][5]{=}u_1, A[1][1][43]{=}u_2, A[2][0][1]{=}A[2][1][1]{=}u_3,$ $A[2][0][2]{=}A[2][1][2]{=}u_4, A[2][0][3]{=}A[2][1][3]{=}u_5, A[2][0][4]{=}A[2][1][4]{=}u_6,$ $A[2][0][5]{=}A[2][1][5]{=}u_7, A[2][0][6]{=}A[2][1][6]{=}u_8, A[2][0][7]{=}A[2][1][7]{=}u_9,$ $A[2][0][9]{=}A[2][1][9]{=}u_{10}, A[2][0][10]{=}A[2][1][10]{=}u_{11}, A[2][0][11]{=}A[2][1][11]{=}u_{12},$ $A[2][0][12]{=}A[2][1][12]{=}u_{13}, A[2][0][13]{=}A[2][1][13]{=}u_{14}, A[2][0][14]{=}A[2][1][14]{=}u_{15},$ $A[2][0][15]{=}A[2][1][15]{=}A[3][0][33]{=}u_{16}, A[2][0][17]{=}A[2][1][17]{=}u_{17},$ $A[2][0][18]{=}A[2][1][18]{=}u_{18}, A[2][0][19]{=}A[2][1][19]{=}u_{19}, A[2][0][20]{=}A[2][1][20]{=}u_{20},$ $A[2][0][21]{=}A[2][1][21]{=}u_{21}, A[2][0][22]{=}A[2][1][22]{=}u_{22}, A[2][0][23]{=}A[2][1][23]{=}u_{23},$ $A[2][0][24]{=}A[2][1][24]{=}u_{24}, A[2][0][25]{=}A[2][1][25]{=}u_{25}, A[2][0][26]{=}A[2][1][26]{=}u_{26},$ $A[2][0][28]{=}A[2][1][28]{=}A[3][0][62]{=}A[3][1][62]{=}u_{27}, A[2][0][30]{=}A[2][1][30]{=}u_{28},$ $A[2][0][31]{=}A[2][1][31]{=}u_{29}, A[2][0][32]{=}A[2][1][32]{=}u_{30}, A[2][0][34]{=}A[2][1][34]{=}u_{31},$ $A[2][0][36]{=}A[2][1][36]{=}u_{32}, A[2][0][37]{=}A[2][1][37]{=}u_{33}, A[2][0][38]{=}A[2][1][38]{=}u_{34},$ $A[2][0][41]{=}A[2][1][41]{=}u_{35}, A[2][0][42]{=}A[2][1][42]{=}u_{36}, A[2][0][43]{=}A[2][1][43]{=}u_{37},$ $A[2][0][44]{=}A[2][1][44]{=}u_{38}, A[2][0][45]{=}A[2][1][45]{=}u_{39}, A[2][0][46]{=}A[2][1][46]{=}u_{40},$ $A[2][0][50]{=}A[2][1][50]{=}u_{41}, A[2][0][51]{=}A[2][1][51]{=}u_{42}, A[2][0][54]{=}A[2][1][54]{=}u_{43},$ $A[2][0][57]{=}A[2][1][57]{=}u_{44}, A[2][0][59]{=}A[2][1][59]{=}u_{45}, A[2][0][60]{=}A[2][1][60]{=}u_{46},$ $A[2][0][61]{=}A[2][1][61]{=}u_{47}, A[2][0][62]{=}A[2][1][62]{=}u_{48}, A[2][0][63]{=}A[2][1][63]{=}u_{49},$ $A[3][0][1]{=}A[3][1][1]{=}u_{50}, A[3][0][15]{=}A[3][1][15]{=}u_{51}, A[3][0][23]{=}A[3][1][23]{=}u_{52},$ $A[3][0][60]{=}A[3][1][60]{=}u_{53}, A[3][0][36]{=}A[3][1][36]{=}u_{54}, A[3][0][40]{=}A[3][1][40]{=}u_{55},$ $A[3][0][42]{=}A[3][1][42]{=}u_{56}, A[3][0][46]{=}A[3][1][46]{=}u_{57}, A[3][0][47]{=}A[3][1][47]{=}u_{58},$ $A[3][0][54]{=}A[3][1][54]{=}u_{59}, A[3][0][55]{=}A[3][1][55]{=}u_{60}, A[3][0][56]{=}A[3][1][56]{=}u_{61},$ $A[3][0][59]{=}A[3][1][59]{=}u_{62}$

Table 7: Parameter sets for attack on 6-round Ketje SR v1

kernel quadratic term
$A[0][2][0]{=}A[0][4][0]{=}A[1][2][9]{=}A[1][4][9]{=}v_0, A[1][3][6]{=}A[1][4][6]{=}A[2][3][4]{=}v_1,$
Bit Condition
$A[4][1][14]{=}k_{39}+k_{70}+k_{119}+n_{63}+n_{94}+n_{143}+n_{159}+n_{174}+n_{223}+1$
Ordinary Cube Variables
$A[1][2][0]{=}A[1][3][0]{=}u_0, A[1][3][2]{=}A[1][4][2]{=}u_1, A[1][3][4]{=}A[1][4][4]{=}u_2,$ $A[1][2][7]{=}A[1][4][7]{=}u_3, A[2][2][7]{=}u_4, A[2][3][7]{=}u_5, A[2][4][7]{=}u_4{+}u_5,$ $A[2][2][9]{=}u_6, A[2][3][9]{=}u_7, A[2][4][9]{=}u_6{+}u_7, A[2][2][10]{=}u_8, A[2][3][10]{=}u_9,$ $A[2][4][10]{=}u_8{+}u_9, A[2][2][12]{=}A[2][3][12]{=}u_{10}, A[2][2][13]{=}A[2][4][13]{=}u_{11},$ $A[4][2][0]{=}u_{12}, A[4][3][0]{=}u_{13}, A[4][4][0]{=}u_{12}{+}u_{13}, A[4][1][1]{=}u_{14}, A[4][2][1]{=}u_{15},$ $A[4][3][1]{=}u_{16}, A[4][4][1]{=}u_{14}{+}u_{15}{+}u_{16}, A[4][1][2]{=}u_{17}, A[4][2][2]{=}u_{18}, A[4][3][2]{=}u_{19},$ $A[4][4][2]{=}u_{17}{+}u_{18}{+}u_{19}, A[4][1][3]{=}u_{20}, A[4][2][3]{=}u_{21}, A[4][3][3]{=}u_{22},$ $A[4][4][3]{=}u_{20}{+}u_{21}{+}u_{22}, A[4][1][6]{=}u_{23}, A[4][2][6]{=}u_{24}, A[4][4][6]{=}u_{23}{+}u_{24},$ $A[4][1][13]{=}A[4][3][13]{=}u_{25}, A[4][2][8]{=}u_{26}, A[4][3][8]{=}u_{27}, A[4][4][8]{=}u_{26}{+}u_{27},$ $A[4][2][9]{=}u_{28}, A[4][3][9]{=}u_{29}, A[4][4][9]{=}u_{28}{+}u_{29}, A[4][1][10]{=}u_{14}{+}u_{15}{+}u_{16},$ $A[4][2][10]{=}u_{30}, A[4][4][10]{=}u_{14}{+}u_{15}{+}u_{16}{+}u_{30}$

Table 8: Parameter sets for attack on 7-round Ketje SR v1

kernel quadratic term
$A[0][2][0]{=}A[0][4][0]{=}A[1][2][9]{=}A[1][4][9]{=}v_0$,$A[1][3][6]{=}A[1][4][6]{=}A[2][3][4]{=}v_1$,
Bit Condition
$A[0][2][1]{=}k_8 + k_{57} + k_{88} + n_{32} + n_{112} + n_{192} + 1$ $A[2][3][5]{=}k_{13} + k_{44} + k_{93} + k_{124}$ $A[4][1][14]{=}k_{39} + k_{70} + k_{119} + n_{94} + n_{143} + n_{159} + n_{174} + 1$ $A[0][3[3]{=}k_{26} + k_{75} + k_{91} + k_{106} + n_{130} + 1$ $A[1][3[13]{=}k_{21} + k_{52} + k_{101} + n_{141}$ $A[1][4][7]{=}k_{15} + k_{46} + k_{95} + k_{111} + k_{126} + 1$ $A[1][2][1]{=}k_9 + k_{40} + k_{89} + k_{120}+ n_{49} + n_{64} + n_{113} + n_{193} + 1$
Ordinary Cube Variables
$A[0][2][3]{=}A[0][4][3]{=}u_0$,$A[0][2][5]{=}A[0][4][5]{=}u_1$,$A[0][2][6]{=}A[0][4][6]{=}u_2$, $A[0][2][8]{=}A[0][4][8]{=}u_3$,$A[0][3][9]{=}A[0][4][9]{=}u_4$,$A[0][2][10]{=}A[0][4][10]{=}u_5$, $A[0][2][12]{=}u_6$,$A[0][3][12]{=}u_7$,$A[0][4][12]{=}u_6{+}u_7$,$A[0][2][13]{=}u_8$,$A[0][3][13]{=}u_9$, $A[0][4][13]{=}u_8{+}u_9$,$A[0][2][14]{=}A[0][4][14]{=}u_{10}$,$A[1][2][2]{=}A[1][4][2]{=}u_{11}$, $A[1][2][5]{=}u_{12}$,$A[1][3][5]{=}u_{13}$,$A[1][4][5]{=}u_{12}{+}u_{13}$,$A[1][2][7]{=}A[1][3][7]{=}u_{14}$, $A[1][2][8]{=}u_{15}$,$A[1][3][8]{=}u_{16}$,$A[1][4][8]{=}u_{15}{+}u_{16}$,$A[1][2][10]{=}u_{17}$, $A[1][3][10]{=}u_{18}$,$A[1][4][10]{=}u_{17}{+}u_{18}$,$A[1][2][11]{=}u_{19}$, $A[1][4][11]{=}u_{20}$,$A[1][2][12]{=}A[1][4][12]{=}u_{21}$,$A[1][2][13]{=}A[1][4][13]{=}u_{22}$, $A[1][2][14]{=}A[1][4][14]{=}u_{23}$,$A[1][2][15]{=}A[1][4][15]{=}u_{24}$,$A[2][3][1]{=}A[2][4][1]{=}u_{25}$, $A[2][2][2]{=}A[2][4][2]{=}u_{26}$,$A[2][2][4]{=}u_{27}$,$A[2][4][4]{=}u_{28}$,$A[2][2][9]{=}A[2][4][9]{=}u_{29}$, $A[2][2][11]{=}u_{30}$,$A[2][3][11]{=}u_{31}$,$A[2][4][11]{=}u_{30}{+}u_{31}$,$A[2][2][12]{=}u_{32}$,$A[2][3][12]{=}u_{33}$, $A[2][4][12]{=}u_{32}{+}u_{33}$,$A[2][2][13]{=}A[2][4][13]{=}u_{34}$,$A[2][3][14]{=}A[2][4][14]{=}u_{35}$, $A[2][2][15]{=}A[2][4][15]{=}u_{36}$,$A[3][3][0]{=}A[3][4][0]{=}u_{37}$,$A[3][2][1]{=}A[3][4][1]{=}u_{38}$, $A[3][2][4]{=}u_{39}$,$A[3][3][4]{=}u_{40}$,$A[3][4][4]{=}u_{39}{+}u_{40}$,$A[3][2][6]{=}u_{41}$,$A[3][3][6]{=}u_{42}$, $A[3][4][6]{=}u_{41}{+}u_{42}$,$A[3][2][7]{=}u_{43}$,$A[3][3][7]{=}u_{44}$,$A[3][4][7]{=}u_{43}{+}u_{44}$, $A[3][2][9]{=}A[3][4][9]{=}u_{45}$,$A[3][2][12]{=}u_{46}$,$A[3][3][12]{=}u_{47}$,$A[3][4][12]{=}u_{46}{+}u_{47}$, $A[3][2][13]{=}u_{48}$,$A[3][3][13]{=}u_4{+}u_{13}{+}u_{27}{+}u_{28}$,$A[3][4][13]{=}u_4{+}u_{13}{+}u_{27}{+}u_{28}{+}u_{48}$, $A[3][2][14]{=}A[3][4][14]{=}u_{50}$,$A[3][2][15]{=}A[3][4][15]{=}u_{51}$,$A[4][2][0]{=}A[4][4][0]{=}u_{52}$, $A[4][1][1]{=}u_{53}$,$A[4][2][1]{=}u_{54}$,$A[4][3][1]{=}u_{55}$,$A[4][4][1]{=}u_{53}{+}u_{54}{+}u_{55}$, $A[4][2][2]{=}A[4][4][2]{=}u_{56}$,$A[4][1][3]{=}A[4][2][3]{=}u_{57}$,$A[4][1][6]{=}u_{58}$,$A[4][2][6]{=}u_{59}$, $A[4][4][6]{=}u_{58}{+}u_{59}$,$A[4][1][10]{=}u_8{+}u_9{+}u_{22}{+}u_{26}{+}u_{43}{+}u_{44}{+}u_{53}{+}u_{54}{+}u_{55}$, $A[4][2][10]{=}u_{60}$,$A[4][4][10]{=}u_8{+}u_9{+}u_{22}{+}u_{26}{+}u_{43}{+}u_{44}{+}u_{53}{+}u_{54}{+}u_{55}{+}u_{60}$, $A[4][1][8]{=}u_{20}$,$A[4][2][8]{=}u_{61}$,$A[4][4][8]{=}u_{20}{+}u_{61}$,$A[4][2][9]{=}u_{62}$,$A[4][3][9]{=}u_{49}$, $A[4][4][9]{=}u_{62}{+}u_{49}$

Table 9: Parameter sets for attack on 6-round Ketje SR v2

kernel quadratic term
$A[2][0][0]{=}A[2][1][0]{=}A[2][3][1]{=}A[2][4][1]{=}v_0,A[1][2][12]{=}A[1][4][12]{=}A[4][0][1]{=}v_1$
Bit Condition
$A[2][4][10]{=}k_{34} + k_{65} + k_{82} + k_{97} + n_{42} + n_{73} + n_{105} + n_{154} + n_{186} + n_{217}$
Ordinary Cube Variables
$A[0][1][0]{=}u_0,A[0][4][0]{=}u_0,A[0][1][1]{=}u_1,A[0][3][1]{=}u_2,A[0][4][1]{=}u_1{+}u_2,$ $A[0][1][7]{=}u_3,A[0][3][7]{=}u_4,A[0][4][7]{=}u_3{+}u_4,A[0][3][8]{=}u_5,A[0][4][8]{=}u_5,$ $A[0][1][10]{=}u_6,A[0][3][10]{=}u_7,A[0][4][10]{=}u_6{+}u_7,A[0][1][11]{=}u_8,A[0][3][11]{=}u_9,$ $A[0][4][11]{=}u_8{+}u_9,A[0][1][15]{=}u_{10},A[0][3][15]{=}u_{11},A[0][4][15]{=}u_{10}{+}u_{11},$ $A[1][0][7]{=}u_{12},A[1][4][7]{=}u_{12},A[1][0][14]{=}u_{13},A[1][2][14]{=}u_{13},A[2][0][2]{=}u_{14},$ $A[2][3][2]{=}u_{15},A[2][4][2]{=}u_{14}{+}u_{15},A[2][0][3]{=}u_{16},A[2][1][3]{=}u_{17},A[2][3][3]{=}u_{18},$ $A[2][4][3]{=}u_{16}{+}u_{17}{+}u_{18},A[2][0][4]{=}u_{19},A[2][3][4]{=}u_{20},A[2][4][4]{=}u_{19}{+}u_{20},$ $A[2][0][6]{=}u_{21},A[2][1][6]{=}u_{22},A[2][3][6]{=}u_{23},A[2][4][6]{=}u_{21}{+}u_{22}{+}u_{23},$ $A[2][0][9]{=}u_{24},A[2][4][9]{=}u_{24},A[2][1][11]{=}u_{25},A[2][4][11]{=}u_{25},A[3][1][12]{=}u_{26},$ $A[3][2][12]{=}u_{27},A[3][4][12]{=}u_{26}{+}u_{27},A[3][1][15]{=}u_{28},A[3][2][15]{=}u_{28},A[4][0][7]{=}u_{29},$ $A[4][2][7]{=}u_{30},A[4][3][7]{=}u_{29}{+}u_{30}$

Table 10: Parameter sets for attack on 7-round Ketje SR v2

kernel quadratic term
$A[2][0][0]$=$A[2][1][0]$=$A[2][3][1]$=$A[2][4][1]$=v_0,$A[1][2][12]$=$A[1][4][12]$=$A[4][0][1]$=v_1
Bit Condition
$A[0][3[5]$=$k_{28}+k_{109}+n_{21}$,$A[1][4][4]$=$k_{27}+k_{108}+n_{116}$,$A[1][2][0]$=$k_{39}+k_{104}$, $A[2][0][9]$=$k_2+k_{33}+k_{114}+n_9+n_{26}+n_{41}+n_{90}+n_{153}$ $A[4][2][14]$=$k_{39}+k_{70}+k_{87}+k_{102}+n_{78}+n_{110}$ $A[4][2][0]$=$k_{25}+k_{56}+k_{88}+n_{33}+n_{49}+n_{64}+n_{96}+n_{177}$ $A[1][4][9]$=$k_1+k_{32}+k_{113}+n_{152}+1$ $A[0][3[3]$=$k_{26}+k_{107}+n_{83}+n_{115}+n_{131}$ $A[4][2][3]$=$k_{28}+k_{59}+k_{91}+n_{52}+n_{67}+n_{99}$ $A[0][3[13]$=$k_5+k_{36}+k_{117}+n_{29}+n_{93}+n_{125}+1$
Ordinary Cube Variables
$A[0][1][0]$=u_0,$A[0][3][0]$=u_1,$A[0][4][0]$=u_0+u_1,$A[0][1][1]$=u_2,$A[0][3][1]$=u_3, $A[0][4][1]$=u_2+u_3,$A[0][3][4]$=u_4,$A[0][4][4]$=u_4,$A[0][1][5]$=u_5,$A[0][4][5]$=u_5, $A[0][1][6]$=u_6,$A[0][4][6]$=u_6,$A[0][1][7]$=u_7,$A[0][3][7]$=u_8,$A[0][4][7]$=u_7+u_8, $A[0][3][8]$=u_9,$A[0][4][8]$=u_9,$A[0][1][9]$=u_{10},$A[0][3][9]$=u_{11},$A[0][4][9]$=$u_{10}+u_{11}$, $A[0][1][10]$=u_{12},$A[0][3][10]$=u_{12},$A[0][1][15]$=u_{13},$A[0][3][15]$=u_{14},$A[0][4][15]$=$u_{13}+u_{14}$, $A[1][2][5]$=u_{15},$A[1][4][5]$=u_{15},$A[1][0][14]$=u_{16},$A[1][2][14]$=u_{17},$A[2][0][2]$=u_{18}, $A[2][1][2]$=u_{19},$A[2][3][2]$=u_{20},$A[2][4][2]$=$u_{18}+u_{19}+u_{20}$,$A[2][0][3]$=u_{21}, $A[2][1][3]$=u_{22},$A[2][3][3]$=u_{23},$A[2][4][3]$=$u_{21}+u_{22}+u_{23}$,$A[2][0][4]$=u_{24}, $A[2][1][4]$=u_{25},$A[2][3][4]$=u_{26},$A[2][4][4]$=$u_{24}+u_{25}+u_{26}$,$A[2][0][5]$=u_{27}, $A[2][3][5]$=u_{28},$A[2][4][5]$=$u_{27}+u_{28}$,$A[2][0][6]$=u_{29},$A[2][1][6]$=u_{30},$A[2][3][6]$=u_{31}, $A[2][4][6]$=$u_{29}+u_{30}+u_{31}$,$A[2][0][7]$=u_{32},$A[2][1][7]$=u_{33},$A[2][4][7]$=$u_{32}+u_{33}$, $A[2][0][8]$=u_{34},$A[2][1][8]$=u_{35},$A[2][4][8]$=$u_{34}+u_{35}$,$A[2][0][10]$=u_{36},$A[2][1][10]$=u_{37}, $A[2][3][10]$=u_{38},$A[2][4][10]$=$u_{36}+u_{37}+u_{38}$,$A[2][1][11]$=u_{39},$A[2][4][11]$=u_{39}, $A[2][0][12]$=u_{40},$A[2][1][12]$=u_{41},$A[2][3][12]$=u_{40},$A[2][4][12]$=u_{41},$A[4][0][12]$=u_{42}, $A[4][3][12]$=u_{42},$A[2][0][13]$=u_{43},$A[2][1][13]$=u_{44},$A[2][3][13]$=u_{45}, $A[2][4][13]$=$u_{43}+u_{44}+u_{45}$,$A[2][0][14]$=u_{46},$A[2][1][14]$=u_{47},$A[2][3][14]$=u_{48}, $A[2][4][14]$=$u_{46}+u_{47}+u_{48}$,$A[2][0][15]$=u_{49},$A[2][1][15]$=u_{50},$A[2][3][15]$=u_{51}, $A[2][4][15]$=$u_{49}+u_{50}+u_{51}$,$A[3][1][2]$=u_{52},$A[3][2][2]$=u_{53},$A[3][4][2]$=$u_{52}+u_{53}$, $A[4][0][4]$=u_{54},$A[4][2][4]$=u_{55},$A[4][3][4]$=$u_{54}+u_{55}$,$A[4][0][5]$=u_{56},$A[4][2][5]$=u_{57}, $A[4][3][5]$=$u_{56}+u_{57}$,$A[4][0][6]$=$u_{49}+u_{51}+u$,$A[4][3][6]$=$u_{49}+u_{51}+u$, $A[4][2][8]$=u_{58},$A[4][3][8]$=u_{58},$A[4][0][7]$=u_{59},$A[4][2][7]$=u_{60},$A[4][3][7]$=$u_{59}+u_{60}$, $A[4][0][13]$=u_{61},$A[4][2][13]$=u_{62},$A[4][3][13]$=$u_{61}+u_{62}$

Table 11: Parameter sets for attack on 9-round `KMAC256`

kernel quadratic term
$A[0][0][0]=A[0][3][0]=A[2][0][2]=A[2][2][2]=v_0, A[1][1][20]=A[3][1][9]=v_1$
Bit Condition
$k_{14} + k_{207} + k_{334} + k_{527} + k_{654} + k_{847} + k_{911} + k_{974} + k_{1167} + k_{1294} + k_{1487} + 1 = 0$
Ordinary Cube Variables
$A[0][0][1]=u_0, A[0][1][1]=u_1, A[0][2][1]=u_2, A[0][3][1]=u_0+u_1+u_2, A[0][0][2]=u_3, A[0][1][2]=u_4, A[0][2][2]=u_5, A[0][3][2]=u_3+u_4+u_5, A[0][0][3]=u_6, A[0][1][3]=u_7, A[0][2][3]=u_8, A[0][3][3]=u_6+u_7+u_8, A[0][0][4]=u_9, A[0][1][4]=u_{10}, A[0][2][4]=u_{11}, A[0][3][4]=u_9+u_{10}+u_{11}, A[0][0][5]=u_{12}, A[0][1][5]=u_{13}, A[0][2][5]=u_{14}, A[0][3][5]=u_{12}+u_{13}+u_{14}, A[0][0][6]=u_{15}, A[0][1][6]=u_{16}, A[0][2][6]=u_{17}, A[0][3][6]=u_{15}+u_{16}+u_{17}, A[0][0][7]=u_{18}, A[0][1][7]=u_{19}, A[0][2][7]=u_{18}+u_{19}, A[0][0][9]=u_{20}, A[0][1][9]=u_{21}, A[0][2][9]=u_{22}, A[0][3][9]=u_{20}+u_{21}+u_{22}, A[0][0][10]=u_{23}, A[0][1][10]=u_{24}, A[0][3][10]=u_{23}+u_{24}, A[0][0][11]=u_{25}, A[0][1][11]=u_{26}, A[0][2][11]=u_{27}, A[0][3][11]=u_{25}+u_{26}+u_{27}, A[0][0][12]=u_{28}, A[0][1][12]=u_{29}, A[0][2][12]=u_{30}, A[0][3][12]=u_{28}+u_{29}+u_{30}, A[0][0][13]=u_{31}, A[0][1][13]=u_{32}, A[0][2][13]=u_{33}, A[0][3][13]=u_{31}+u_{32}+u_{33}, A[0][0][21]=u_{34}, A[0][1][21]=u_{35}, A[0][2][21]=u_{36}, A[0][3][21]=u_{34}+u_{35}+u_{36}, A[0][1][23]=u_{37}, A[0][2][23]=u_{38}, A[0][3][23]=u_{37}+u_{38}, A[0][0][26]=u_{39}, A[0][3][26]=u_{39}, A[0][0][27]=u_{40}, A[0][1][27]=u_{41}, A[0][2][27]=u_{42}, A[0][3][27]=u_{40}+u_{41}+u_{42}, A[0][0][33]=u_{43}, A[0][1][33]=u_{44}, A[0][2][33]=u_{45}, A[0][3][33]=u_{43}+u_{44}+u_{45}, A[0][0][35]=u_{46}, A[0][1][35]=u_{47}, A[0][2][35]=u_{48}, A[0][3][35]=u_{46}+u_{47}+u_{48}, A[0][0][43]=u_{49}, A[0][1][43]=u_{50}, A[0][2][43]=u_{51}, A[0][3][43]=u_{49}+u_{50}+u_{51}, A[0][0][45]=u_{52}, A[0][1][45]=u_{53}, A[0][2][45]=u_{54}, A[0][3][45]=u_{52}+u_{53}+u_{54}, A[0][0][46]=u_{55}, A[0][1][46]=u_{56}, A[0][2][46]=u_{57}, A[0][3][46]=u_{55}+u_{56}+u_{57}, A[0][0][47]=u_{58}, A[0][1][47]=u_{59}, A[0][2][47]=u_{60}, A[0][3][47]=u_{58}+u_{59}+u_{60}, A[0][0][49]=u_{61}, A[0][1][49]=u_{62}, A[0][2][49]=u_{63}, A[0][3][49]=u_{61}+u_{62}+u_{63}, A[0][0][53]=u_{64}, A[0][1][53]=u_{65}, A[0][2][53]=u_{66}, A[0][3][53]=u_{64}+u_{65}+u_{66}, A[0][0][57]=u_{67}, A[0][1][57]=u_{68}, A[0][2][57]=u_{69}, A[0][3][57]=u_{67}+u_{68}+u_{69}, A[0][1][60]=u_{70}, A[0][2][60]=u_{71}, A[0][3][60]=u_{70}+u_{71}, A[0][0][61]=u_{72}, A[0][1][61]=u_{73}, A[0][2][61]=u_{74}, A[0][3][61]=u_{72}+u_{73}+u_{74}, A[0][0][63]=u_{75}, A[0][1][63]=u_{76}, A[0][2][63]=u_{77}, A[0][3][63]=u_{75}+u_{76}+u_{77}, A[1][0][6]=u_{78}, A[1][1][6]=u_{79}, A[1][2][6]=u_{78}+u_{79}, A[1][0][16]=u_{80}, A[1][1][16]=u_{81}, A[1][2][16]=u_{80}+u_{81}, A[1][0][26]=u_{82}, A[1][2][26]=u_{82}, A[1][0][43]=u_{83}, A[1][1][43]=u_{84}, A[1][2][43]=u_{83}+u_{84}, A[1][0][52]=u_{85}, A[1][1][52]=u_{86}, A[1][2][52]=u_{85}+u_{86}, A[1][0][57]=u_{87}, A[1][1][57]=u_{88}, A[1][2][57]=u_{87}+u_{88}, A[1][0][62]=u_{89}, A[1][1][62]=u_{90}, A[1][2][62]=u_{89}+u_{90}, A[2][0][10]=u_{91}, A[2][2][10]=u_{91}, A[2][0][18]=u_{92}, A[2][1][18]=u_{93}, A[2][2][18]=u_{92}+u_{93}, A[2][0][22]=u_{94}, A[2][1][22]=u_{95}, A[2][2][22]=u_{94}+u_{95}, A[2][0][23]=u_{96}, A[2][1][23]=u_{97}, A[2][2][23]=u_{96}+u_{97}, A[2][0][25]=u_{98}, A[2][1][25]=u_{99}, A[2][2][25]=u_{98}+u_{99}, A[2][0][32]=u_{100}, A[2][1][32]=u_{101}, A[2][2][32]=u_{100}+u_{101}, A[2][0][42]=u_{102}, A[2][1][42]=u_{103}, A[2][2][42]=u_{102}+u_{103}, A[2][0][55]=u_{104}, A[2][1][55]=u_{105}, A[2][2][55]=u_{104}+u_{105}, A[3][0][2]=u_{106}, A[3][1][2]=u_{107}, A[3][2][2]=u_{106}+u_{107}, A[3][0][8]=u_{108}, A[3][1][8]=u_{109}, A[3][2][8]=u_{108}+u_{109}, A[3][0][24]=u_{110}, A[3][1][24]=u_{111}, A[3][2][24]=u_{110}+u_{111}, A[3][0][33]=u_{112}, A[3][1][33]=u_{113}, A[3][2][33]=u_{112}+u_{113}, A[3][0][42]=u_{114}, A[3][1][42]=u_{115}, A[3][2][42]=u_{114}+u_{115}, A[3][0][44]=u_{116}, A[3][1][44]=u_{117}, A[3][2][44]=u_{116}+u_{117}, A[3][0][46]=u_{118}, A[3][1][46]=u_{119}, A[3][2][46]=u_{118}+u_{119}, A[3][0][48]=u_{120}, A[3][1][48]=u_{121}, A[3][2][48]=u_{120}+u_{121}, A[3][0][51]=u_{122}, A[3][1][51]=u_{123}, A[3][2][51]=u_{122}+u_{123}, A[3][0][57]=u_{124}, A[3][2][57]=u_{124}, A[4][0][31]=u_{125}, A[4][1][31]=u_{126}, A[4][2][31]=u_{125}+u_{126}$