Tight Security Analysis of EHtM MAC

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Outline of the talk

- Definition and Security Game of MAC.
- Hash-then-Mask.
- Enhanced Hash-then-Mask
- Forgery Attack on Enhanced Hash-then-Mask
- Sketch of Security Proof
- Summary

Categories of MAC: Stateful or Probabilistic

$\Pi = (\mathrm{KG}, \mathrm{TG}, \mathrm{Ver})$ is a triplet of algorithms

- KG is called key-generation algorithm that outputs a key $K \xleftarrow{\$} \mathcal{K}$ (Key-space).
- $\mathrm{TG} : \mathcal{K} \times \mathcal{IV} \times \mathcal{M} \rightarrow \mathcal{T}$ is called tag generation algorithm.
- $\mathrm{Ver} : \mathcal{K} \times \mathcal{IV} \times \mathcal{M} \times \mathcal{T} \rightarrow \{0, 1\}$ such that

$$\mathrm{Ver}(K, IV, M, T) = \begin{cases} 1 & \text{if } \mathrm{TG}(K, IV, M) = T, \\ 0 & \text{otherwise} \end{cases}$$

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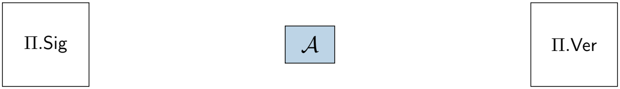
- Stateful : IV is a counter / nonce (e.g XMACC, PCS)
- Probabilistic : IV is random (e.g XMACR, EHtM)

Security Game of Probabilistic MAC



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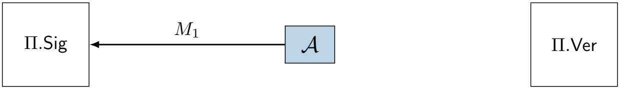

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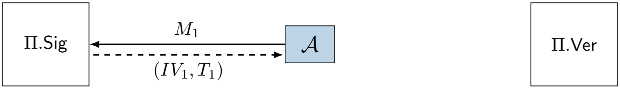

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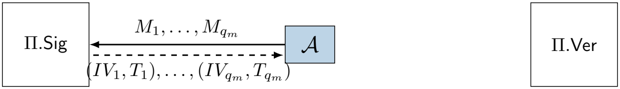

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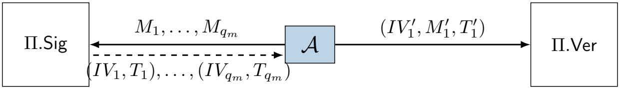

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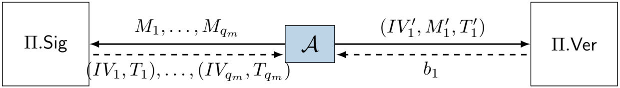

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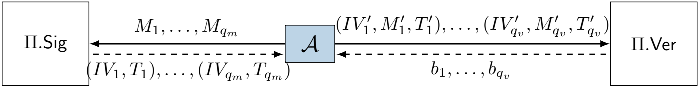

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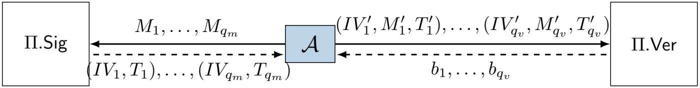

Verification queries can be interleaved with MAC queries and should be fresh.

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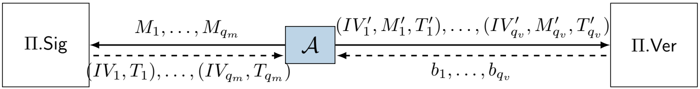

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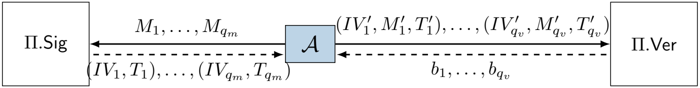

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$\mathbf{Adv}_{\Pi}^{\mathrm{mac}}(\mathcal{A}) = \Pr[\exists i : b_i = 1]$.

Π is secure against all such computationally bounded adversary $\mathcal{A}$, if the probability of obtaining $b_i = 1$ for any $i \in \{1, \ldots, q_v\}$ is small.

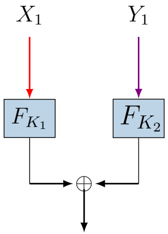
Birthday And Beyond the Birthday Bound (BBB) Security

- Birthday Bound: Security is void after $2^{n/2}$ queries (e.g CBC-MAC, LightMAC)

- Drawback: Not practical when block size is small (e.g PRINCE, HEIGHT, LED etc.)

- Beyond Birthday Bound: Security remains even after $2^{n/2}$ queries ("**Beyond Birthday Bound Security**") without increasing the output length (e,g SUM-ECBC, PMAC_Plus, 3kf9).

Abstraction: Sum function is not a PRF



$$T_1 = F_{K_1}(X_1) \oplus F_{K_2}(Y_1)$$

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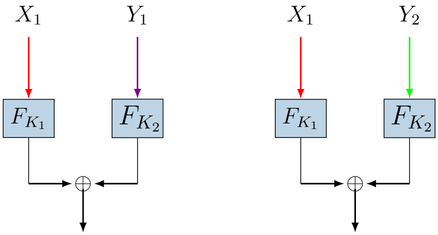

$T_1 = F_{K_1}(X_1) \oplus F_{K_2}(Y_1)$ $\quad$ $T_2 = F_{K_1}(X_1) \oplus F_{K_2}(Y_2)$

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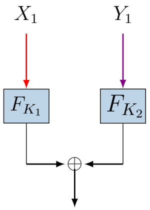

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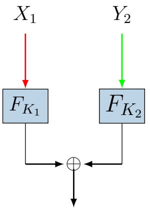

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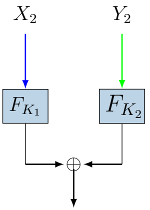

$T_3 = F_{K_1}(X_2) \oplus F_{K_2}(Y_2)$

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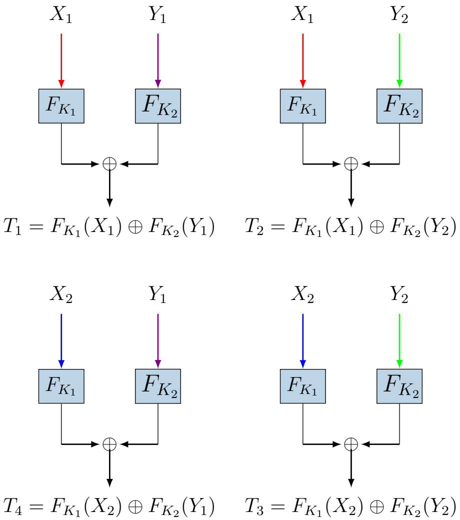

Alternating Cycle (AC)

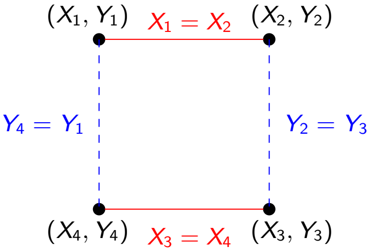

Figure : Alternating Cycle (AC) of length 4

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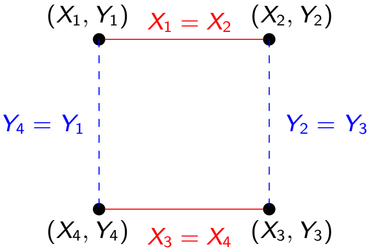
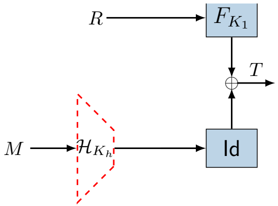


Figure : Alternating Cycle (AC) of length 4

AC in the input of sum function makes the sum of its output zero, i.e. $T_1 \oplus T_2 \oplus T_3 \oplus T_4 = 0$.

Attacks on most of the Probabilistic MAC is based on the formation of Alternating Cycle!

Hash-then-Mask (HtM): First instantiation of Probabilistic MAC

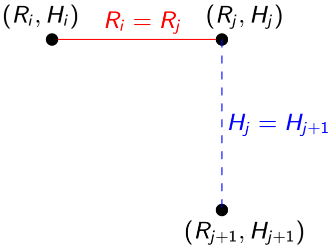


- Id is the identity function.
- Birthday bound security and the bound is also tight.

Birthday Attack of HtM

(R_i, H_i) $\quad R_i = R_j \quad$ (R_j, H_j)

Birthday Attack of HtM



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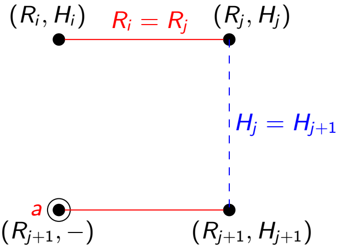

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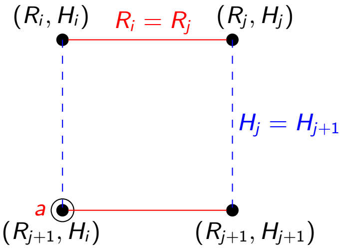

For valid verification attempt, we need to set T_a to $T_i \oplus T_j \oplus T_{j+1}$.

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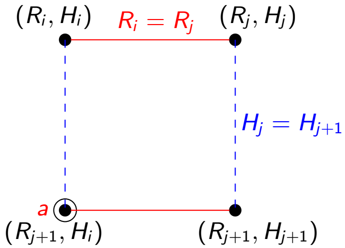

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Query Complexity

If we make roughly $2^{n/2}$ many MAC queries, then the top right edge holds w.h.p

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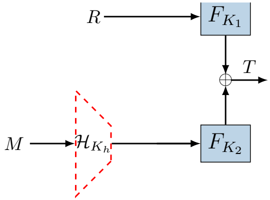
How can we beat the birthday barrier ? (Replacing Id with F_{K_2} !)



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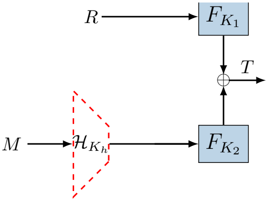

- The previous attack for Hash-then-Mask works here.

Existing BBB Secure Probabilistic MAC.

MAC	Randomness	PRF	Security Model
MACRX_3 [CRYPTO 99]	$3n$	n	Standard
RWMAC [FSE 2010]	n	$2n$ to n	Standard
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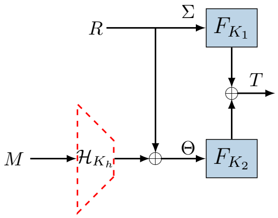
Can we design a Probabilistic MAC with n-bit PRF and n-bit randomness with BBB security in standard model?

Enhanced Hash-then-Mask: Minematsu, FSE 2010

Enhanced Hash-then-Mask (EHtM) by Minematsu, FSE 2010

- EHtM is the first BBB secure (i.e. $2^{2n/3}$-MAC security) probabilistic MAC with *n*-bit PRF and *n*-bit randomness.



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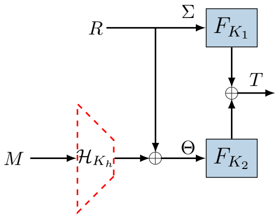

Contribution

We have improved the MAC security bound of EHtM to $2^{3n/4}$ and shown that the bound is tight.

The Fundamental Result.

If $(\widetilde{\Sigma}, \widetilde{\Theta})$ does not contain any alternating cycle, then the distribution of $F_{K_1}(\Sigma_i) \oplus F_{K_2}(\Theta_i)$ is perfectly random.

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We use this result to mount the attack with $2^{3n/4}$ MAC queries, i.e. we'll try to form an alternating cycle!

$2^{3n/4}$ Forging Complexity of EHtM

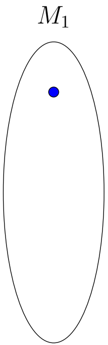
Forging attack is based on the formation of AC4 that consists of two phases:

Part I: Estimation of Hash Difference.

Part II: Forging Attempt for Fixed Estimated Hash Difference.

$2^{3n/4}$ Forging Complexity of EHtM

Part I : Estimation of Hash Difference.



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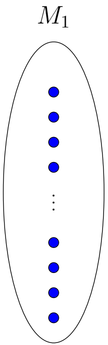

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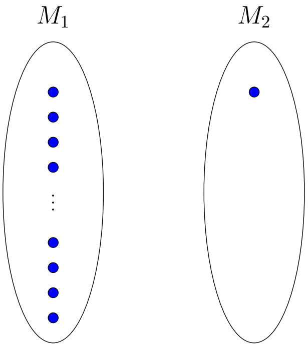

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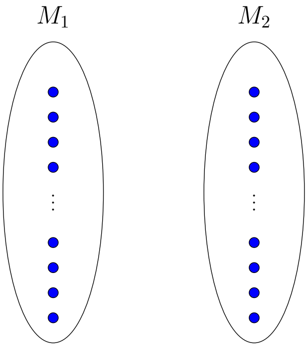

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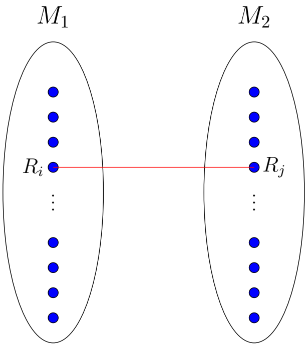

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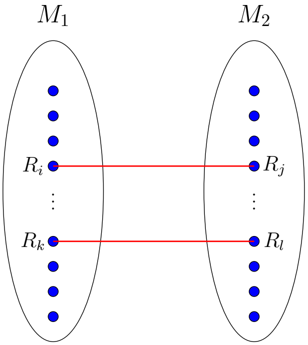

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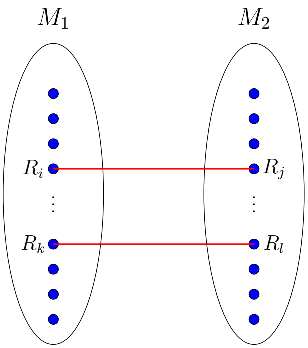

If $T_i \oplus T_j \oplus T_k \oplus T_l = 0$ then, $F_{K_2}(R_i \oplus H(M_1)) \oplus F_{K_2}(R_j \oplus H(M_2)) \oplus F_{K_2}(R_k \oplus H(M_1)) \oplus F_{K_2}(R_l \oplus H(M_2)) = 0$

$2^{3n/4}$ Forging Complexity of EHtM

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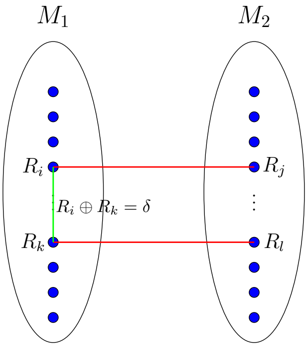
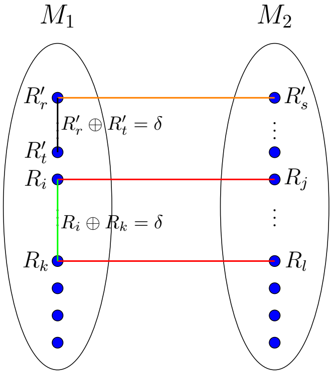


Compute $R_i \oplus R_k = \delta$; estimated hash difference. (False positives may also occur)

$2^{3n/4}$ Forging Complexity of EHtM

Part II : Forging Attempt for a correct guess of δ.



Verification query : $(R'_t, M_2, T'_r \oplus T'_s \oplus T'_t)$.

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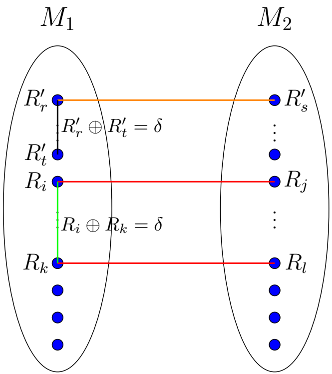

Verification query : $(R'_t, M_2, T'_r \oplus T'_s \oplus T'_t)$.

Attack Complexity is $2^{3n/4}$.

Security Result of EHtM

Theorem

EHtM *is secure upto* $\Theta(q_m^4/2^{3n} + q_v/2^n)$

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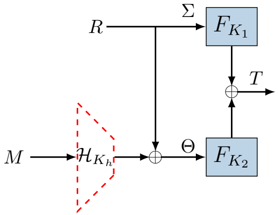
Security Result of EHtM

Theorem

EHtM *is secure upto* $\Theta(q_m^4/2^{3n} + q_v/2^n)$

- Lower bound is already proved using the previous attack ✓
- Now we show the upper bound in subsequent slides
  - We prove using Coefficients-H Technique.
  - For this, we identify the set of bad transcripts (or bad events).
  - Realizing a good transcript is almost as likely as real and the ideal world.

Security Proof of EHtM



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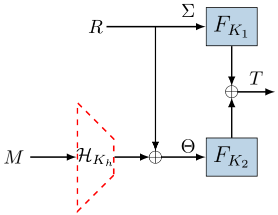

Key Point of Bad Events

Avoid alternating cycles in $(\widetilde{\Sigma}, \widetilde{\Theta})$.

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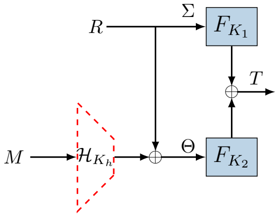
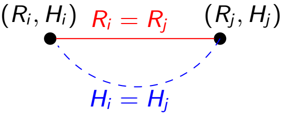


Key Point of Bad Events

Avoid alternating cycles in $(\widetilde{\Sigma}, \widetilde{\Theta})$.

If there is no alternating cycle in $(\widetilde{\Sigma}, \widetilde{\Theta})$ then the output of EHtM is perfectly random.

Security Proof of EHtM: Bad Events



Prob: $q_m^2 \epsilon / 2^n$

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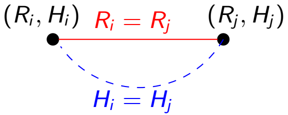

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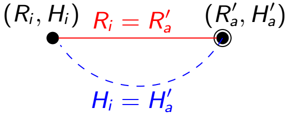

Prob: $q_m^2 / 2^{2n+1} + q_v \epsilon$

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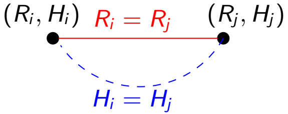

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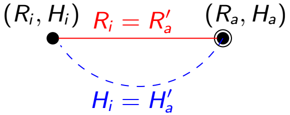

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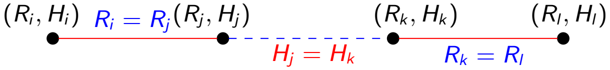
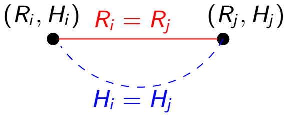
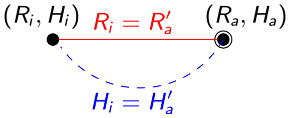


Prob: $q_m^4 \epsilon / 2^{2n}$

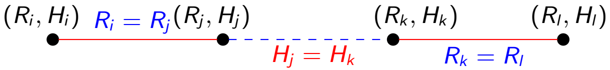
Security Proof of EHtM: Bad Events



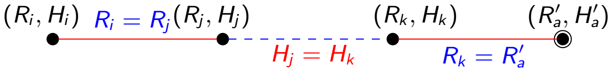
Prob: $q_m^2 \epsilon / 2^n$



Prob: $q_m^2 / 2^{2n+1} + q_v \epsilon$



Prob: $q_m^4 \epsilon / 2^{2n}$



Prob: $q_m^4 / 2^{3n} + q_v \epsilon$

Summary

Till date, EHtM is the best probabilistic MAC in terms of offering security with n-bit randomness and n-bit primitive.

Open Problem

Can we design a probabilistic MAC with n-bit randomness and n-bit primitive that offers optimal security ?

Thank You for your Attention!