A Security Analysis of Deoxys and its Internal Tweakable Block Ciphers

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Outlines

Outline

Deoxys

- A third-round candidate of the CAESAR competition
- Designed by Jérémy Jean, Ivica Nikolić, Thomas Peyrin, Yannick Seurin
- Two AEAD modes:
  - Deoxys-I, the nonce-respecting mode
  - Deoxys-II, the nonce-misuse resistant mode
- Deoxys-BC: AES-based tweakable block cipher
  - Deoxys-BC-256, 14 rounds
  - Deoxys-BC-384, 16 rounds

Deoxys-BC

- AES round function
  - AddRoundTweakey
  - SubBytes
  - ShiftRows
  - MixColumns
- TWEAKEY framework

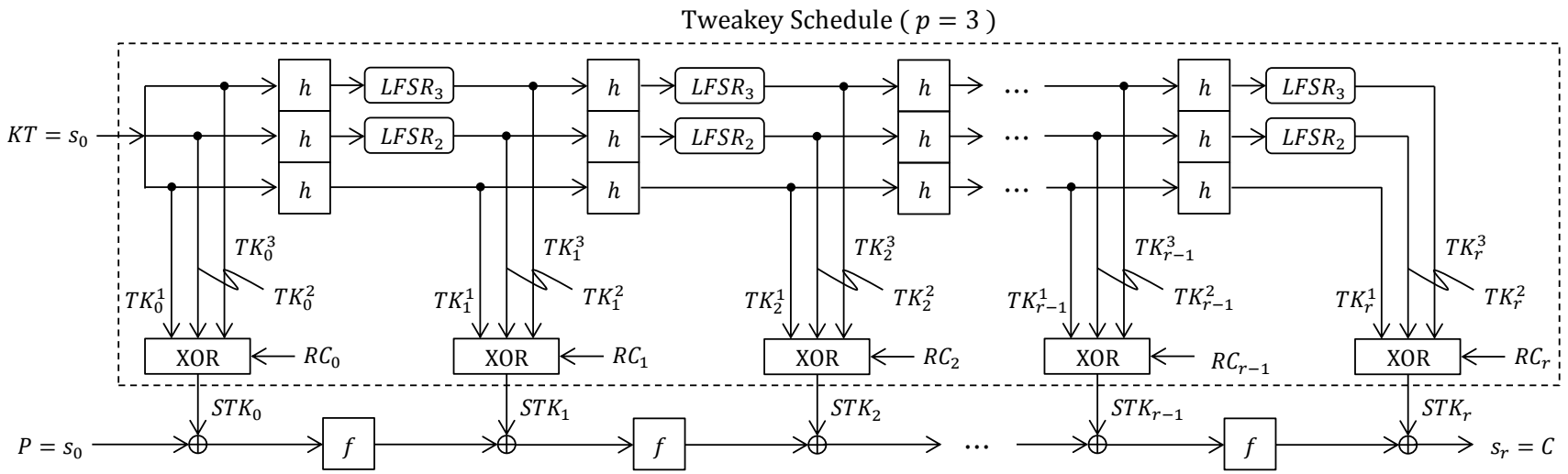


Figure: Instantiation of the TWEAKEY framework for Deoxys-BC-384.

Deoxys-BC

- Sub-tweakeys
  - Deoxys-BC-256: $STK_i = TK_i^1 \oplus TK_i^2 \oplus RC_i$
  - Deoxys-BC-384: $STK_i = TK_i^1 \oplus TK_i^2 \oplus TK_i^3 \oplus RC_i$
- Update of TK
  - $TK_{i+1}^1 = h(TK_i^1)$, $TK_{i+1}^2 = h(LFSR_2(TK_i^2))$, $TK_{i+1}^3 = h(LFSR_3(TK_i^3))$
  - Byte permutation h

$$\begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 1 & 6 & 11 & 12 & 5 & 10 & 15 & 0 & 9 & 14 & 3 & 4 & 13 & 2 & 7 & 8 \end{pmatrix}$$

  - LFSRs

$LFSR_2$	$(x_7 \|\| x_6 \|\| x_5 \|\| x_4 \|\| x_3 \|\| x_2 \|\| x_1 \|\| x_0) \rightarrow (x_6 \|\| x_5 \|\| x_4 \|\| x_3 \|\| x_2 \|\| x_1 \|\| x_0 \|\| x_7 \oplus x_5)$
$LFSR_3$	$(x_7 \|\| x_6 \|\| x_5 \|\| x_4 \|\| x_3 \|\| x_2 \|\| x_1 \|\| x_0) \rightarrow (x_0 \oplus x_6 \|\| x_7 \|\| x_6 \|\| x_5 \|\| x_4 \|\| x_3 \|\| x_2 \|\| x_1)$

Main Results

- New lower bounds on the number of active S-boxes

Deoxys-BC-256

lower bounds	1	2	3	4	5	6	7	8	9	10	11	12	13	14
[JNPS16]	0	0	1	5	9	12	16	17	-	**22**	-	-	-	-
simple model	0	0	1	5	9	12	16	19	23	**26**	29	32	35	38
incompatibility	0	0	1	5	10	14	18	22	27	**31**	35	40	44	48

Deoxys-BC-384

lower bounds	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
[JNPS16]	0	0	0	1	4	8	-	-	-	-	-	**22**	-	-	-	-
simple model	0	0	0	1	4	8	10	14	18	21	24	**28**	31	35	37	45
incompatibility	0	0	0	1	5	9	13	18	22	27	31	**35**	40	44	48	52

Main Results

- Attacks on Deoxys-BC and Deoxys

Deoxys **internal primitives**

	number of rounds	tweak size	key size	time	data	memory	attack type	ref.
Deoxys-BC-256	8/14	128	128	$\leq 2^{128}$	-	-	MitM	[JNPS16]
	$\leq$ 8/14	128	128	$\leq 2^{128}$	-	-	differential	[JNPS16]
	9/14	128	128	2^{118}	2^{117}	2^{117}	rectangle	this
	10/14	$t < 52$	$k > 204$	2^{204}	$2^{127.58}$	$2^{127.58}$	rectangle	this
Deoxys-BC-384	8/16	128	256	$\leq 2^{256}$	-	-	MitM	[JNPS16]
	12/16	128	256	2^{127}	2^{127}	2^{125}	rectangle	this
	13/16	$t < 114$	$k > 270$	2^{270}	2^{127}	2^{144}	rectangle	this

Deoxys **AE schemes**

Deoxys-I-128-128	9/14	-	128	2^{118}	2^{117}	2^{117}	rectangle	this
Deoxys-II-128-128	-	-	128	-	-	-	-	-
Deoxys-I-256-128	12/16	-	256	2^{236}	2^{126}	2^{124}	rectangle	this
Deoxys-II-256-128	-	-	256	-	-	-	-	-

Outline

Single-Key for AES

- For each round, one defines 16 variables $x_i \in \{0, 1\}$, where

$$x_i = \begin{cases} 1, & \text{the } i\text{-th byte is active;} \\ 0, & \text{the } i\text{-th byte is inactive.} \end{cases}$$

- Incorporate the property of branch number 5 of MixColumns:
  Suppose $(x_0, x_5, x_{10}, x_{15}) \xrightarrow{\text{MixColumns}} (x_{16}, x_{17}, x_{18}, x_{19})$

$$x_0 + x_5 + x_{10} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \geq 5d_j,$$
$$d \geq x_0,\ d \geq x_5,\ d \geq x_{10},\ d \geq x_{15},\ d \geq x_{16},\ d \geq x_{17},\ d \geq x_{18},\ d \geq x_{19}.$$

- The objective function:

"minimise Σx_i."

Related-Tweakey with TK^1

- Define 16 variables $stk_i \in \{0, 1\}$, where

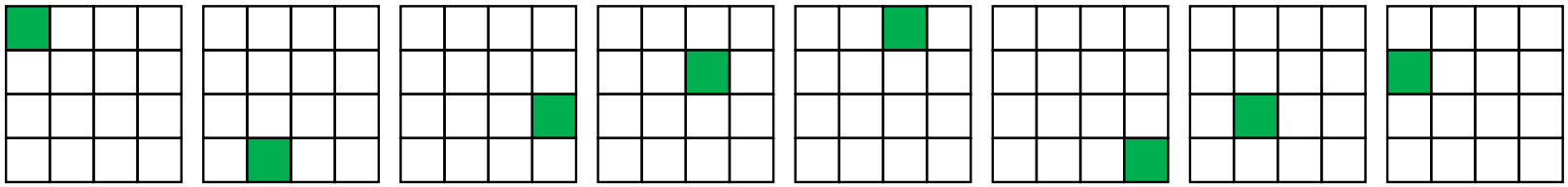
$$stk_i = \begin{cases} 1, & \text{the } i\text{-th subtweakey byte is active;} \\ 0, & \text{the } i\text{-th subtweakey byte is inactive.} \end{cases}$$

- Related-tweakey with TK^1
  - Exclude $(x_i, stk_i, y_i) \in \{(0, 0, 1), (0, 1, 0), (1, 0, 0)\}$ with

$$x_i + stk_i - y_i \geq 0, \qquad x_i - stk_i + y_i \geq 0, \qquad -x_i + stk_i + y_i \geq 0.$$

Related-Tweakey with TK^2 and TK^3

- Differential cancellations may happen.
  - For TK^2, there is at most **1** cancellation for each active byte.
  - For TK^3, there are at most **2** cancellations for each active byte.



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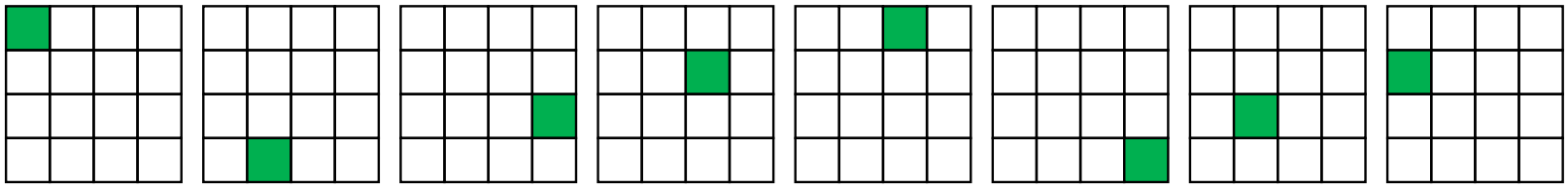


- Let h_{inv} be the inverse of h.

$$\mathrm{LANE}_i - stk_i \geq 0,\ \mathrm{LANE}_i - stk_{16+h_{inv}(i)} \geq 0,\ \cdots,\ \mathrm{LANE}_i - stk_{16(r-1)+h_{inv}^{r-1}(i)} \geq 0,$$
$$stk_i + stk_{16+h_{inv}(i)} + stk_{32+h_{inv}^2(i)} + \cdots + stk_{16(r-1)+h_{inv}^{r-1}(i)} \geq r \cdot \mathrm{LANE}_i - \mathbf{1}.$$

or

$$\mathrm{LANE}_i - stk_i \geq 0,\ \mathrm{LANE}_i - stk_{16+h_{inv}(i)} \geq 0,\ \cdots,\ \mathrm{LANE}_i - stk_{16(r-1)+h_{inv}^{r-1}(i)} \geq 0,$$
$$stk_i + stk_{16+h_{inv}(i)} + stk_{32+h_{inv}^2(i)} + \cdots + stk_{16(r-1)+h_{inv}^{r-1}(i)} \geq r \cdot \mathrm{LANE}_i - \mathbf{2}.$$

Application of the Simple Model

- New lower bounds on the number of active S-boxes

Deoxys-BC-256

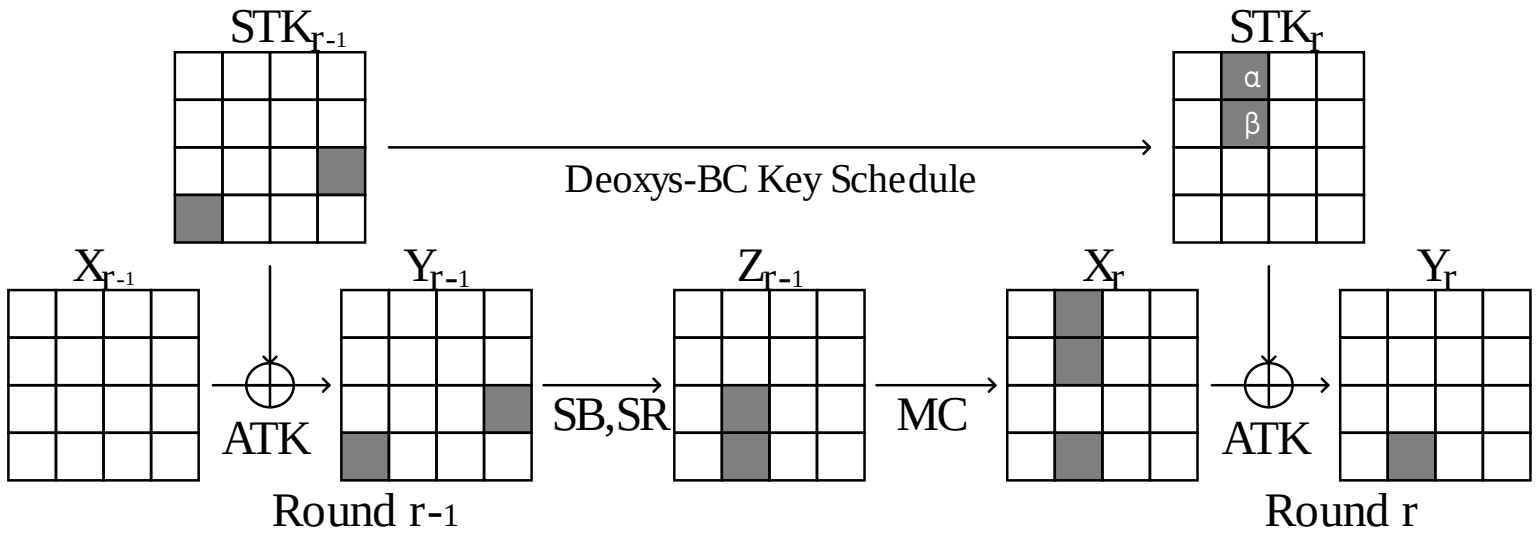
lower bounds	1	2	3	4	5	6	7	8	9	10	11	12	13	14
[JNPS16]	0	0	1	5	9	12	16	17	-	**22**	-	-	-	-
simple model	0	0	1	5	9	12	16	19	23	**26**	29	32	35	38

Deoxys-BC-384

lower bounds	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
[JNPS16]	0	0	0	1	4	8	-	-	-	-	-	**22**	-	-	-	-
simple model	0	0	0	1	4	8	10	14	18	21	24	**28**	31	35	37	45

Limitation of the Simple Model

- There may exist linear incompatibilities.
- Difference cancellations between STK and the state imposes some linear relation of key bytes.
  - E.g., $0xF2 \cdot \alpha + 0xF6 \cdot \beta = 0$



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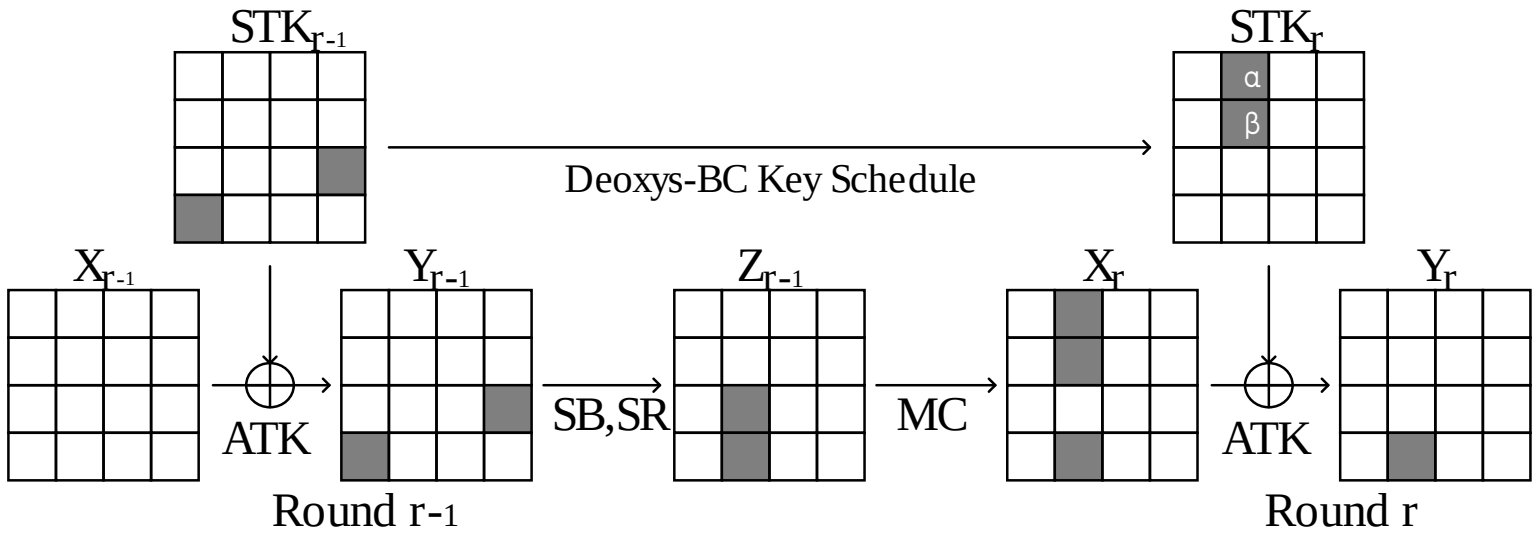


  - Cost additional $b + c - a$ bytes of degree of freedom

    a: Number of active bytes before MC. E.g., $a = 2$

    b: Number of inactive bytes after MC. E.g., $b = 1$

    c: Number of cancellations in ATK. E.g., $c = 2$

Degrees of Freedom

- Degrees of freedom available
  - $s \cdot \sum \mathrm{LANE}_i$
  - $s = 2$ for TK^2 and $s = 3$ for TK^3
- Degrees of consumption

  Type 1 Cancellations in STK,
  - $TK^1[i] \oplus TK^2[i] = 0$ or $TK^1[i] \oplus TK^2[i] \oplus TK^3[i] = 0$

  Type 2 Cancellations between STK and the state
  - Consume $b + c - a$ bytes of degree of freedom

Representation with MILP

- Degrees of consumption Type 1 for r rounds

$$r \cdot \sum_{i=0}^{15} \mathrm{LANE}_i - \sum_{i=0}^{16r-1} stk_i$$

- Degrees of consumption Type 2:

  Suppose that $(x_0, x_5, x_{10}, x_{15}) \xrightarrow{MC} (x_{16}, x_{17}, x_{18}, x_{19})$

  - $a = x_0 + x_5 + x_{10} + x_{15}$
  - $b = 4d - x_{16} - x_{17} - x_{18} - x_{19}$ where $d = 1$ means the column is active.
  - For each byte of the column (x_i, stk_i, y_i)

$$-x_i - stk_i + y_i + c_i \geq -1, \quad x_i + stk_i + y_i - c_i \geq 0,$$
$$-x_i - stk_i - y_i - c_i \geq -3, \; -x_i + stk_i - y_i - c_i \geq -2, \quad x_i - stk_i - y_i - c_i \geq -2.$$

  - $b + c - a$

$$4d - x_{16} - x_{17} - x_{18} - x_{19} + (c_{16} + c_{17} + c_{18} + c_{19}) - (x_0 + x_5 + x_{10} + x_{15}).$$

Representation in the MILP model

- Total consumption of degrees

$$s \cdot \sum_{i=0}^{15} \mathrm{LANE}_i \geq \left(r \cdot \sum_{i=0}^{15} \mathrm{LANE}_i - \sum_{i=0}^{16r-1} stk_i \right) + \sum_{j=0}^{4r-1} \mathrm{TYPE2}_j.$$

- New lower bounds on the number of active S-boxes

`Deoxys-BC-256`

lower bounds	1	2	3	4	5	6	7	8	9	10	11	12	13	14
[JNPS16]	0	0	1	5	9	12	16	17	-	**22**	-	-	-	-
simple model	0	0	1	5	9	12	16	19	23	**26**	29	32	35	38
incompatibility	0	0	1	5	10	14	18	22	27	**31**	35	40	44	48

`Deoxys-BC-384`

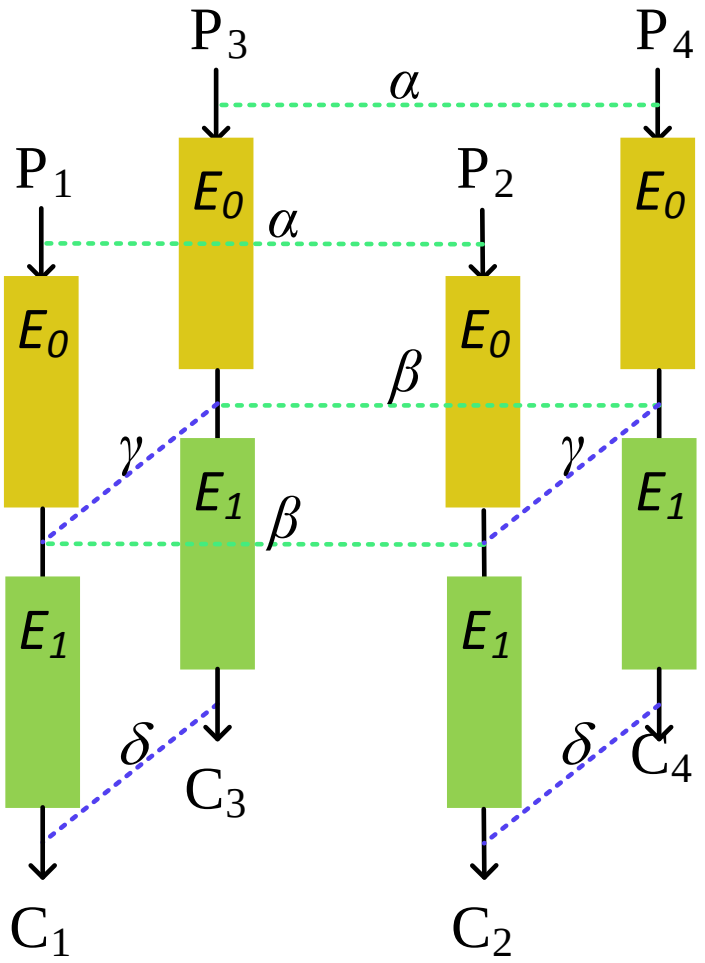
lower bounds	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
[JNPS16]	0	0	0	1	4	8	-	-	-	-	-	**22**	-	-	-	-
simple model	0	0	0	1	4	8	10	14	18	21	24	**28**	31	35	37	45
incompatibility†	0	0	0	1	5	9	13	18	22	27	31	**35**	40	44	48	52

†Bounds for linear incompatibility are obtained under certain assumptions.

Outline

Introduction of Boomerang attacks

- $E = E_1 \circ E_0$
- Two trails $\alpha \xrightarrow{E_0} \beta$, $\gamma \xrightarrow{E_1} \delta$ with probabilities p and q respectively
- A right quartet can be obtained with probability p^2q^2
  - Choose P_1, $P_2 = P_1 \oplus \alpha$
  - $C_1 = E(P_1), C_2 = E(P_2)$
  - Let $C_3 = C_1 \oplus \delta, C_4 = C_2 \oplus \delta$
  - $P_3 = E^{-1}(C_3), P_4 = E^{-1}(C_4)$
  - Test $P_3 \oplus P_4 = \alpha$



Boomerang Switch

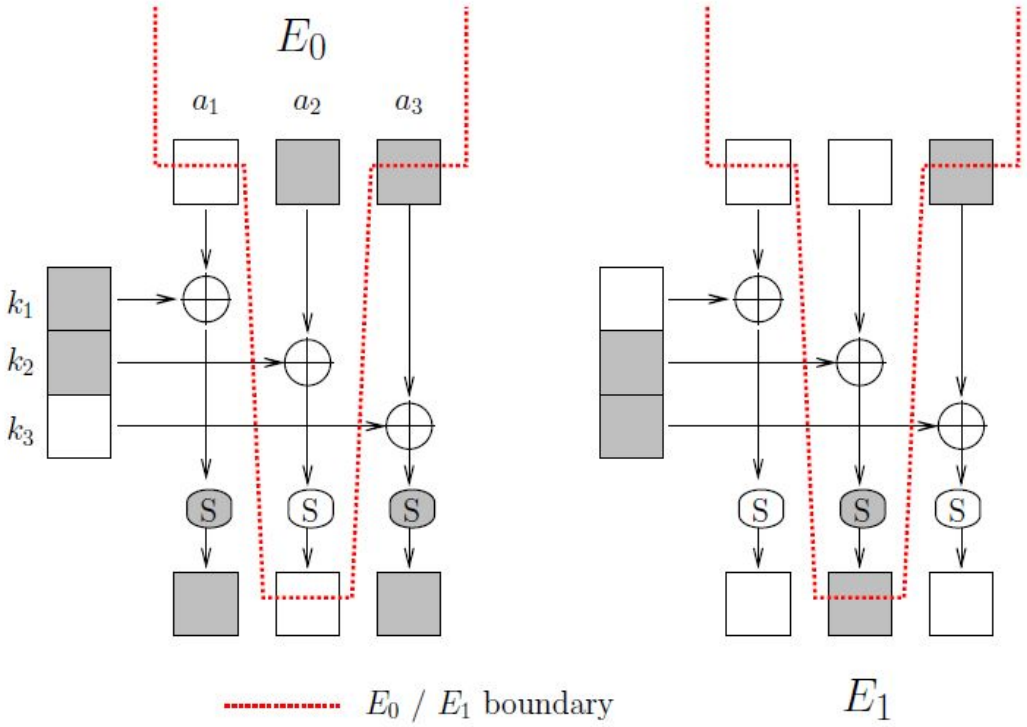


Figure: The ladder switch in a toy three S-box block [BK09].

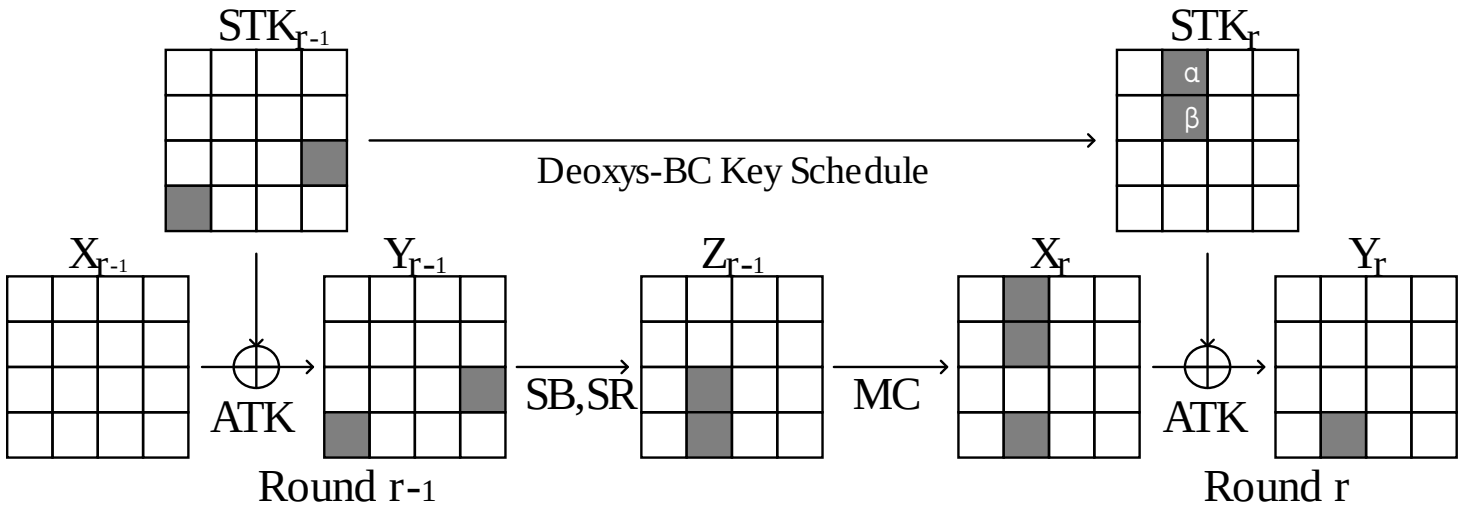
An Example of the Boomerang Switch

10-round distinguisher of Deoxys-BC-384

R	X	K	Y	Z	p_r
5	00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00	69 00 00 00 00 bb 00 00 00 00 d2 00 00 00 00 69	69 00 00 00 00 bb 00 00 00 00 d2 00 00 00 00 69	** 00 00 00 ** 00 00 00 ** 00 00 00 ** 00 00 00	1
6	** 00 00 00 ** 00 00 00 ** 00 00 00 ** 00 00 00	00 10 00 00 00 9e 00 00 00 8e 00 00 00 8e 00 00	** 10 00 00 ** 9e 00 00 ** 8e 00 00 ** 8e 00 00	** ** 00 00 ** 00 00 ** 00 00 ** ** 00 ** ** 00	1
5	00 ** ** ** ** 00 ** ** ** ** 00 ** ** ** ** **	00 ee 00 00 00 00 00 00 00 00 00 00 00 00 00 11	00 ** ** ** ** 00 ** ** ** ** 00 ** ** ** ** 00	00 ** ** ** 00 ** ** ** 00 ** ** ** 00 ** ** **	1
6	00 00 00 00 00 9e 00 00 00 0a ab 00 00 00 93 7a	00 00 00 00 00 00 00 00 00 0a 00 00 00 00 93 00	00 00 00 00 00 9e 00 00 00 00 ab 00 00 00 00 7a	00 00 00 00 68 00 00 00 01 00 00 00 b9 00 00 00	2^{-6}

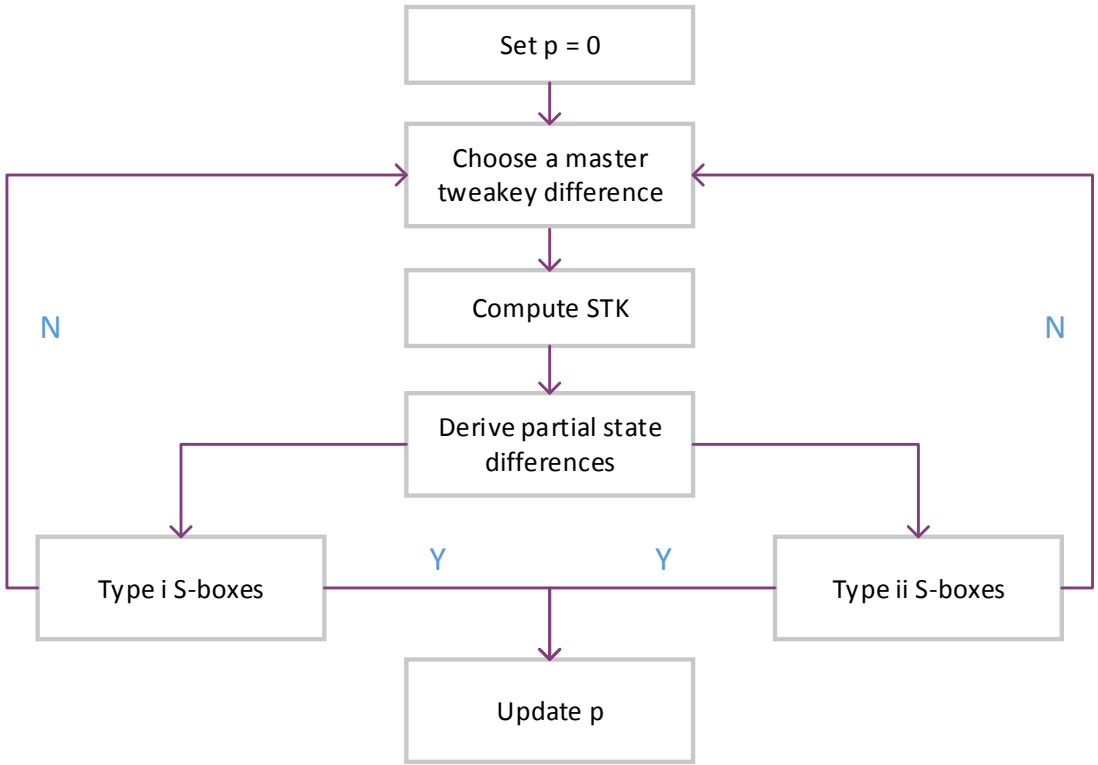
Properties of Truncated Differential Trails

- A few degrees of freedom are left for the master tweakey difference.
- Once the master tweakey difference is fixed, many active bytes of the state are also fixed.



Search for Differential Trails

- Define two types of S-box
  - Type i the input and output differences are determined.
  - Type ii the input or output differences are not determined but some constraints are imposed by the subtweakey differences.
- Given a truncated differential trail



Boomerang Distinguishers

Deoxys-BC-256				Deoxys-BC-384			
R_1, R_2	#AS	pq	$\hat{p}^2\hat{q}^2$	R_1, R_2	#AS	pq	$\hat{p}^2\hat{q}^2$
4,4	6	2^{-36}	2^{-72}	5,5	4	2^{-24}	2^{-42}
5,4	9	2^{-61}	2^{-122}	6,5	9	2^{-60}	2^{-120}
5,5	16	2^{-106}	2^{-212}	6,6	15	2^{-98}	2^{-196}
6,5	20	2^{-136}	2^{-265}	7,6	20	2^{-134}	2^{-268}

Boomerang Attacks

Deoxys **internal primitives**

	number of rounds	tweak size	key size	time	data	memory	attack type	ref.
Deoxys-BC-256	8/14	128	128	$\leq 2^{128}$	-	-	MitM	[JNPS16]
	$\leq$ 8/14	128	128	$\leq 2^{128}$	-	-	differential	[JNPS16]
	9/14	128	128	2^{118}	2^{117}	2^{117}	rectangle	this
	10/14	$t < 52$	$k > 204$	2^{204}	$2^{127.58}$	$2^{127.58}$	rectangle	this
Deoxys-BC-384	8/16	128	256	$\leq 2^{256}$	-	-	MitM	[JNPS16]
	12/16	128	256	2^{127}	2^{127}	2^{125}	rectangle	this
	13/16	$t < 114$	$k > 270$	2^{270}	2^{127}	2^{144}	rectangle	this

Deoxys **AE schemes**

Deoxys-I-128-128	9/14	-	128	2^{118}	2^{117}	2^{117}	rectangle	this
Deoxys-II-128-128	-	-	128	-	-	-	-	-
Deoxys-I-256-128	12/16	-	256	2^{236}	2^{126}	2^{124}	rectangle	this
Deoxys-II-256-128	-	-	256	-	-	-	-	-

Outline

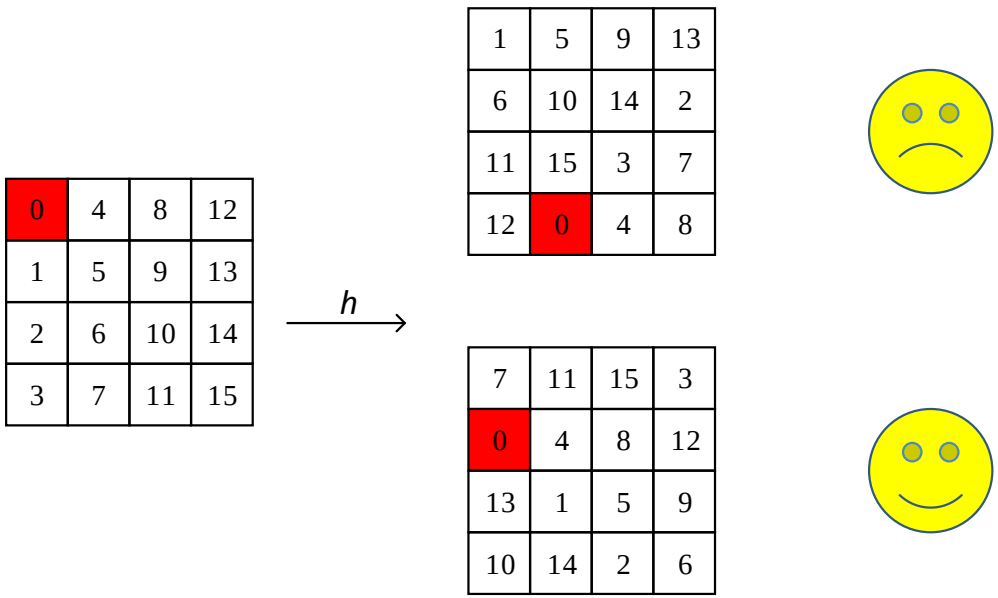
Conclusion

- Two improved lower bounds for the number of active S-boxes for Deoxys-BC under the related-tweakey setting
- Algorithm for searching exact differential trails for Deoxys-BC
- Improved attacks on Deoxys-BC and Deoxys

A Misunderstanding

Byte permutation h in the Tweakey Schedule

$$\begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 1 & 6 & 11 & 12 & 5 & 10 & 15 & 0 & 9 & 14 & 3 & 4 & 13 & 2 & 7 & 8 \end{pmatrix}$$



Thank you for your attention!

Thank all the group members at ASK 2016 for fruitful discussion.