# Rotational-XOR Cryptanalysis of Reduced-round SPECK

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[SGL+17] S. Sun, D. Gerault, P. Lafourcade, Q. Yang, Y. Todo, K. Qiao, and L. Hu. Analysis of AES, Skinny, and others with constraint programming. ToSC 2017 [SHW+14] S. Sun, L. Hu, P. Wang, K. Qiao, X. Ma, and L. Song. Automatic security evaluation and (related-key) differential characteristic search: Application to SIMON, PRESENT, LBlock, DES (L) and other bit-oriented block ciphers. ASIACRYPT 2014 [ST17] Y. Sasaki, and Y. Todo. New impossible differential search tool from design and cryptanalysis aspects. EUROCRYPT 2017

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- Challenge: find an efficient method to encode the cryptographic problem

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#### RX-difference v2

The RX-difference of a pair  $(x_1, x_2)$ :

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Given an RX-difference  $\delta$ , an RX-pair is  $(x, (x \ll \gamma) \oplus \delta)$ .

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$$X = x \oplus a_1$$
$$Y = y \oplus b_1$$
$$Z = X \boxplus Y$$
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#### RX-difference propagation in modular addition

Assume that input RX-differences are  $d_x, d_y, \, {\rm output} \, {\rm RX-difference}$  is  $d_z.$  Then,

$$\begin{aligned} &\Pr[(d_x, d_y) \to d_z] = \\ &1_{(I \oplus SHL)(\delta_x \oplus \delta_y \oplus \delta_z) \oplus 1 \preceq SHL((\delta_x \oplus \delta_z)|(\delta_y \oplus \delta_z))} \cdot 2^{-|SHL((\delta_x \oplus \delta_z)|(\delta_y \oplus \delta_z))|} \cdot 2^{-3} \\ &+ 1_{(I \oplus SHL)(\delta_x \oplus \delta_y \oplus \delta_z) \preceq SHL((\delta_x \oplus \delta_z)|(\delta_y \oplus \delta_z))} \cdot 2^{-|SHL((\delta_x \oplus \delta_z)|(\delta_y \oplus \delta_z))|} \cdot 2^{-1.415}, \end{aligned}$$

where

$$\delta_x = L'(d_x), \delta_y = L'(d_y), \delta_z = L'(d_z).$$





Search for RX-characteristics in the key part and data part

SMT file – Modular Addition

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Condition 1

 $(I \oplus SHL)((\Delta_1 a^r \gg \alpha) \oplus \Delta_1 b^r \oplus \Delta_1 d^r) \oplus 1$  $\leq SHL(((\Delta_1 a^r \gg \alpha) \oplus \Delta_1 d^r)|(\Delta_1 b^r \oplus \Delta_1 d^r))$  $w_r = |SHL(((\Delta_1 a^r \gg \alpha) \oplus \Delta_1 d^r)|(\Delta_1 b^r \oplus \Delta_1 d^r))| + 3$ 

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$$(I \oplus SHL)((\Delta_1 a^r \gg \alpha) \oplus \Delta_1 b^r \oplus \Delta_1 d^r) \leq SHL(((\Delta_1 a^r \gg \alpha) \oplus \Delta_1 d^r)|(\Delta_1 b^r \oplus \Delta_1 d^r)) w_r = |SHL(((\Delta_1 a^r \gg \alpha) \oplus \Delta_1 d^r)|(\Delta_1 b^r \oplus \Delta_1 d^r))| + 1.415$$

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Total weight of a characteristic  $W_{data} = \sum_{r} w_{r}$ 

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SMT file – Objective functions

 $\min W_{data}$ 

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- Other strategy?

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Binary search

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- Terminate after the red interval collapsed

RX-characteristics found in SPECK

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Version	Rounds	Data Prob.	Key Class Size	Ref.
32/64	9	$2^{-30}$	$2^{64}$	[Din14]
32/64	10	$2^{-19.15}$	$2^{28.10}$	This paper
32/64	11	$2^{-22.15}$	$2^{18.68}$	This paper
32/64	12	$2^{-25.57}$	$2^{4.92}$	This paper
48/96	10	$2^{-40}$	$2^{96}$	[Din14]
48/96	11	$2^{-45}$	$2^{96}$	[FWG+ 16]
48/96	11	$2^{-24.15}$	$2^{25.68}$	This paper
48/96	11	$2^{-23.15}$	$2^{14.93}$	This paper
48/96	12	$2^{-26.57}$	$2^{43.51}$	This paper
48/96	13	$2^{-31.98}$	$2^{24.51}$	This paper
48/96	14	$2^{-37.40}$	$2^{0.34}$	This paper
48/96	15	$2^{-43.81}$	$2^{1.09}$	This paper

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Version	Rounds	Data Prob.	Key Class Size	Ref.
64/128	14	$2^{-60}$	$2^{128}$	[Din14]
64/128	15	$2^{-62}$	$2^{128}$	[FWG+16]
64/128	13	$2^{-37.98}$	$2^{21.92}$	This paper
96/144	13	$2^{-84}$	$2^{144}$	[Din14]
96/144	16	$2^{-87}$	$2^{144}$	[FWG+16]
96/144	13	$2^{-37.98}$	$2^{37.92}$	This paper
128/256	14	$2^{-112}$	$2^{256}$	[Din14]
128/256	19	$2^{-119}$	$2^{256}$	[FWG+16]
128/256	13	$2^{-31.98}$	$2^{182.51}$	This paper

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# Thank You!