

Grøstl₅₁₂ Distinguishing Attack: A New Rebound Attack of an AES-like Permutation

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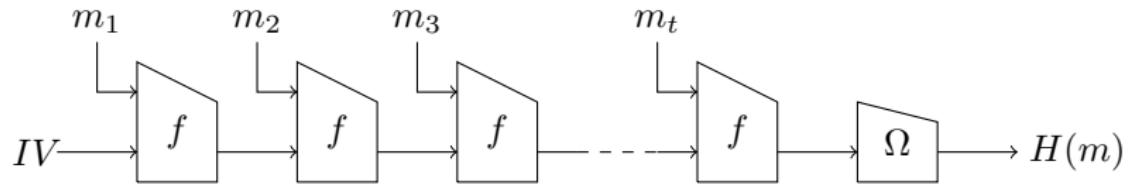
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FSE 2018

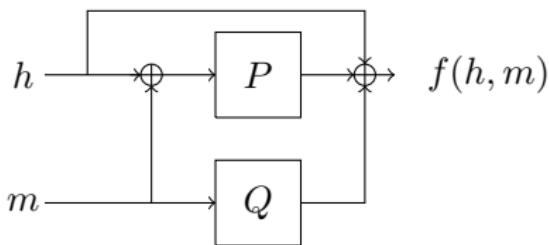
- 1 Grøstl₅₁₂ hash function
- 2 10-round Rebound Attack on Grøstl₅₁₂ Permutations
- 3 11-round Rebound Attack on Grøstl₅₁₂ Permutations

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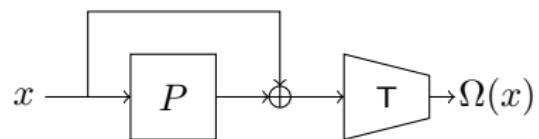
Grøstl₅₁₂ Mode of Operation



Grøstl₅₁₂ internal functions

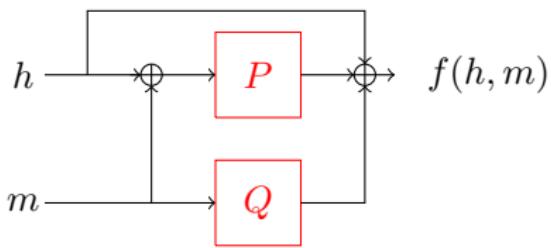


The compression function f

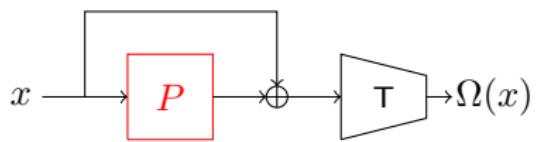


The output transformation Ω

Grøstl₅₁₂ internal functions



The compression function f



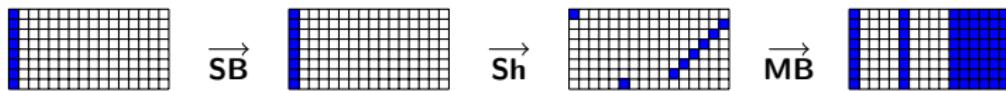
The output transformation Ω

Grøstl₅₁₂ security assertion

- P and Q ideal $\Rightarrow f$ collision and preimage resistant [FSZ09].
- P and Q ideal, independant \Rightarrow Grøstl₅₁₂ indifferentiable from a random oracle [AMP10].

Grøstl₅₁₂ inner permutation P

14 iterations of the following round function:



- 1 Grøstl₅₁₂ hash function
- 2 10-round Rebound Attack on Grøstl₅₁₂ Permutations
- 3 11-round Rebound Attack on Grøstl₅₁₂ Permutations

Limited-birthday distinguishers

Problem

Limited-birthday(P, E_{in}, E_{out}): Given a permutation P and two \mathbb{F}_2 -linear subspaces E_{in} and E_{out} , find a pair of input values (X, X') such that $X \oplus X' \in E_{in}$ and $P(X) \oplus P(X') \in E_{out}$.

Theorem (Gilbert,Peyrin in [GP10])

For a n -bit permutation P , a \mathbb{F}_2 -subspace E_{in} of dimension d_i , a \mathbb{F}_2 -subspace E_{out} of dimension d_o and $d_i \leq d_o$, the computational complexity \mathcal{C}_{gen} of the generic limited-birthday algorithm solving **Limited-birthday**(P, E_{in}, E_{out}) satisfies:

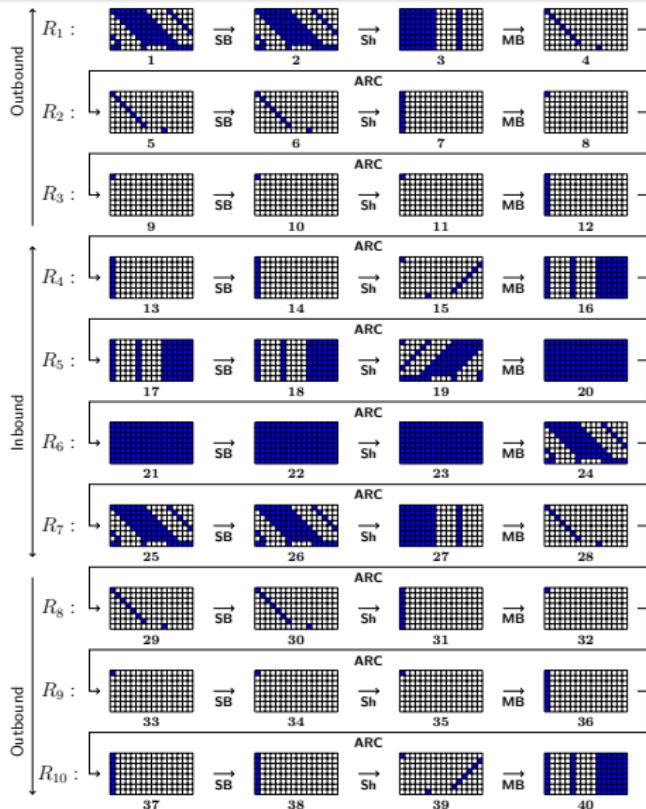
$$\log_2(\mathcal{C}_{gen}) = \begin{cases} (n - d_o)/2 & \text{if } n < 2d_i + d_o, \\ n - d_i - d_o & \text{otherwise.} \end{cases}$$

Optimality has been proven by Iwamoto, Peyrin and Sasaki in [IPS13].

Goals of a Rebound Attack

- Find E_{in} and E_{out} such that there exist an algorithm which solves **Limited-birthday**(P, E_{in}, E_{out}) faster than the generic algorithm.
- The assumption on which the security proof of the hash function relies on is not valid anymore.
- Some rebound attack may be used to mount collision attacks [MRST09].

10-round truncated differential path [Jea13]

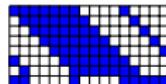


Ways of a Rebound Attack

- **Inbound phase:** Collect many samples designed to satisfy 4 middle rounds of the truncated differential path. Find couples of state values compatible with 2 differentials δ_{in} and δ_{out} propagated respectively forward and backward.
- **Outbound phase:** Find among those couples of state values one satisfying both probabilistic transitions towards the first and last rounds.

Generic limited-birthday algorithm complexity

- Initial state:



$$\dim(E_{in}) = 64 \cdot 8$$

- Final state:



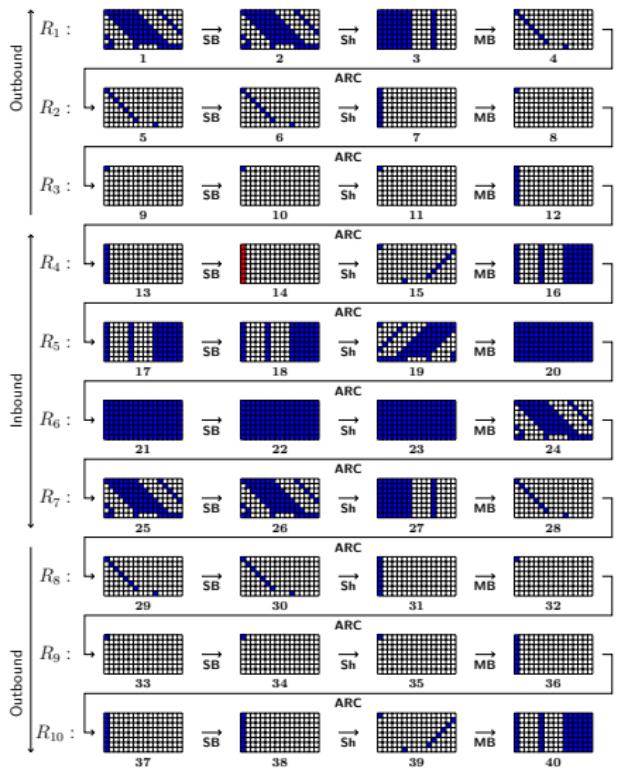
$$\dim(E_{out}) = 8 \cdot 8$$

- Computational complexity:

$$\log_2(\mathcal{C}_{gen}) = (128 - 64 - 8) \cdot 8 = 56 \cdot 8 = 448$$

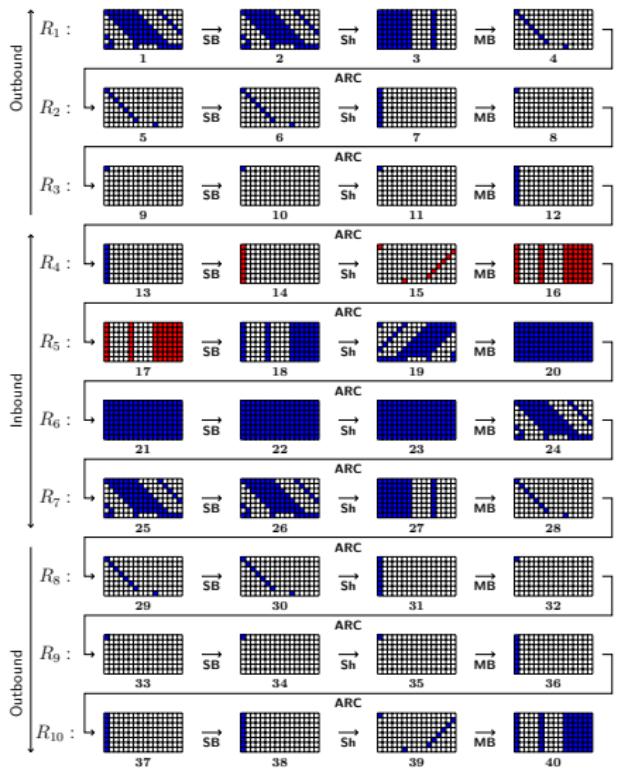
Selection of a differential δ_{in}

- Choose $\delta_{in} \in P_{14}$.
 $(2^{8 \cdot 8} \text{ elements})$



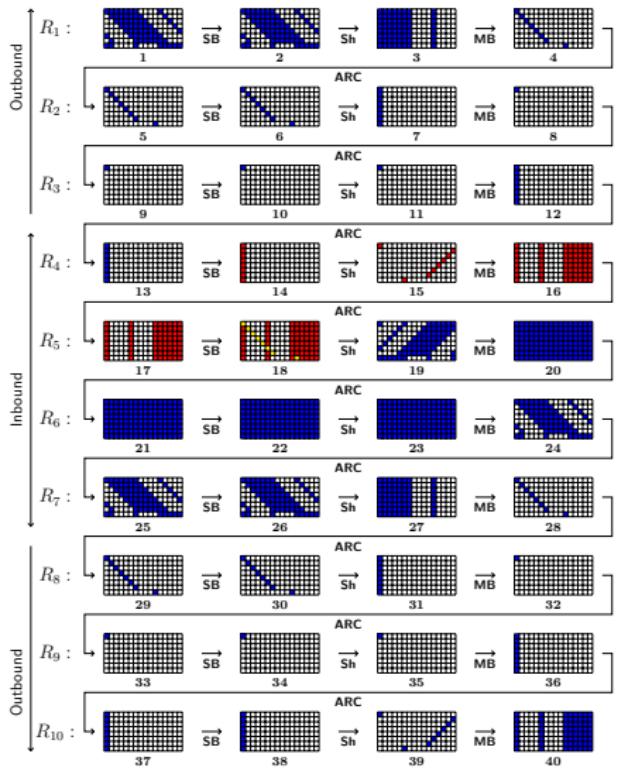
Deterministic propagation of δ_{in}

- Choose $\delta_{in} \in P_{14}$.
- $\delta'_{in} = \text{ARC} \circ \text{MB} \circ \text{Sh}(\delta_{in})$.



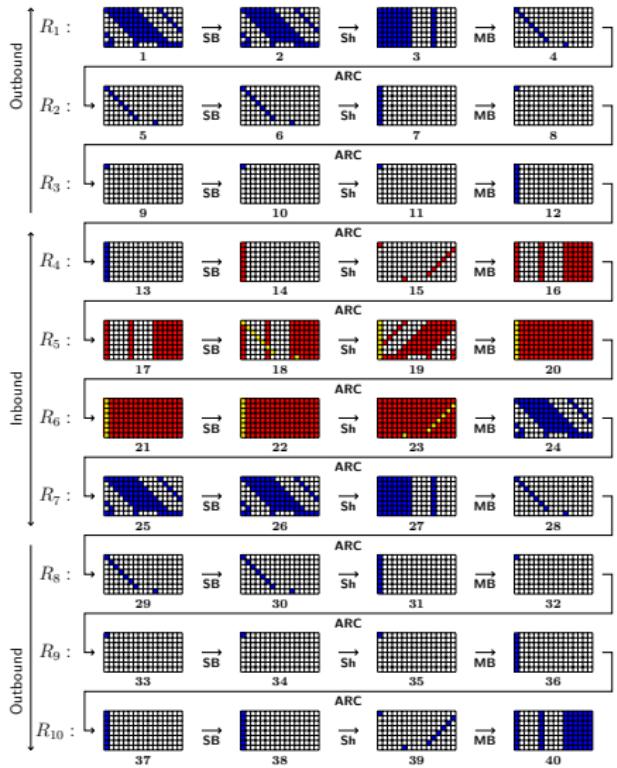
Computation of the 16 lists L_i

- Choose $\delta_{in} \in P_{14}$.
- $\delta'_{in} = \text{ARC} \circ \text{MB} \circ \text{Sh}(\delta_{in})$.
- $L_i = \{(X, X \oplus \delta'_{in}), X \in \mathbb{F}_2^{8 \cdot 8}\}$.



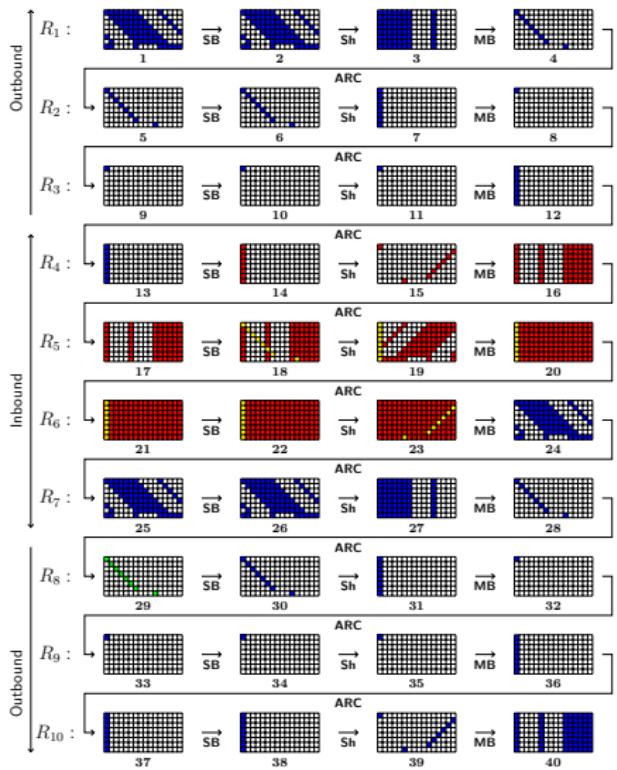
Deterministic propagation of lists L_i

- Choose $\delta_{in} \in P_{14}$.
- $\delta'_{in} = \text{ARC} \circ \text{MB} \circ \text{Sh}(\delta_{in})$.
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- $L'_i = \text{Sh} \circ \text{SB} \circ \mathbf{R}_5(L_i)$.



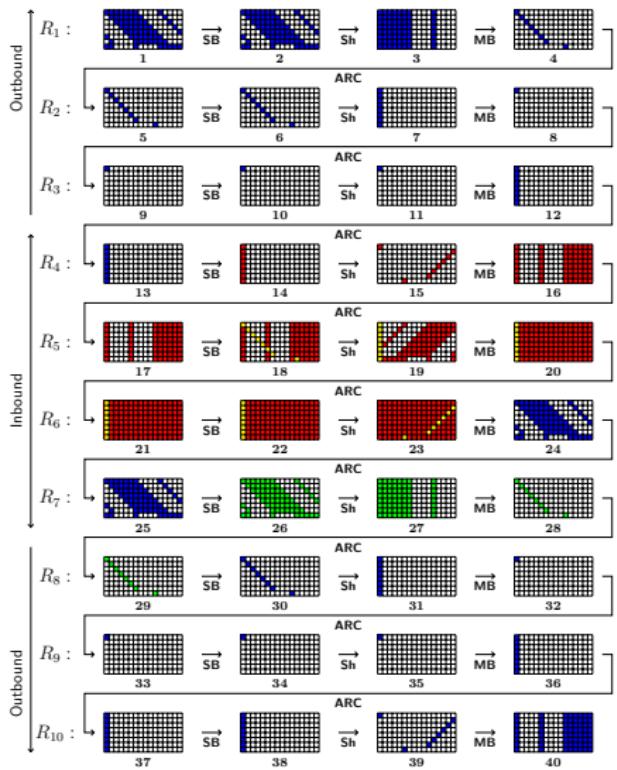
Selection of a differential δ_{out}

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- Choose $\delta_{out} \in P_{29}$.
 $(2^{8 \cdot 8} \text{ elements})$



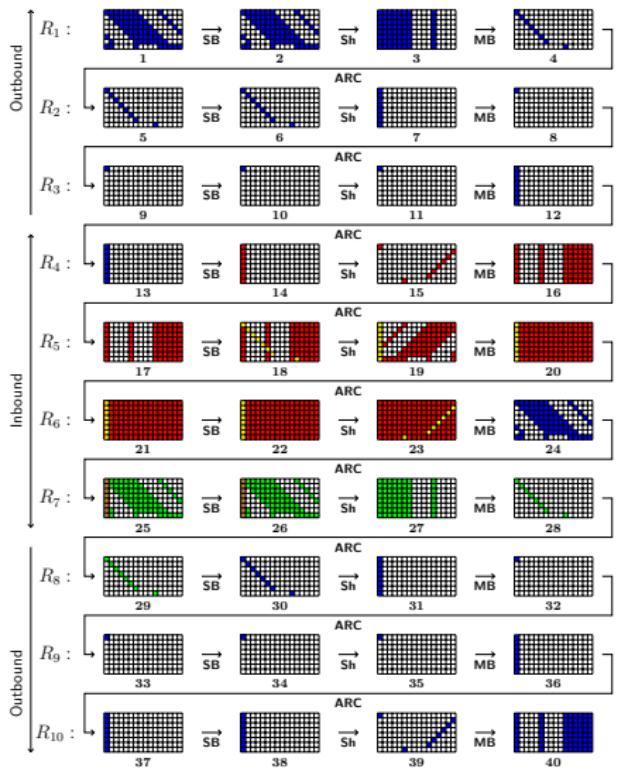
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- $L'_i = \text{Sh} \circ \text{SB} \circ \text{R}_5(L_i)$.
- Choose $\delta_{out} \in P_{29}$.
- $\delta'_{out} = (\text{ARC} \circ \text{MB} \circ \text{Sh})^{-1}(\delta_{out})$.



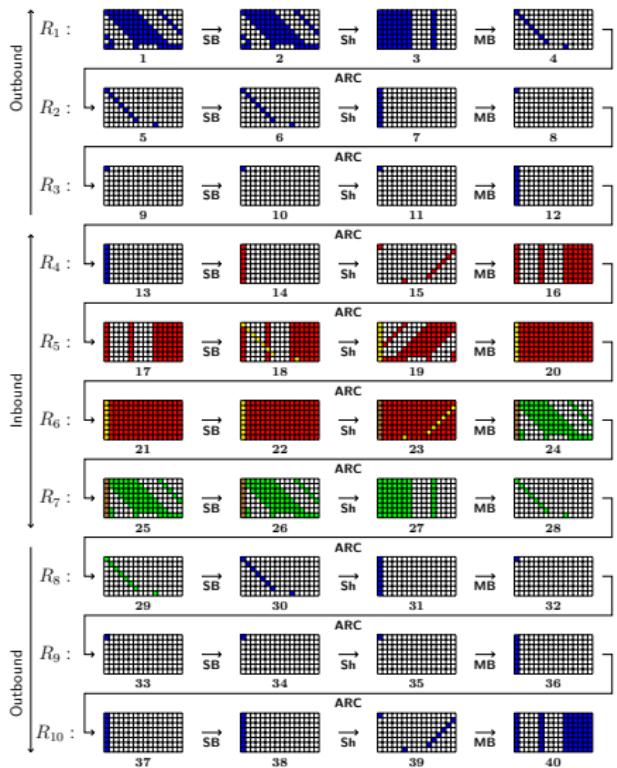
Computation of the 16 lists R_i

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- Choose $\delta_{out} \in P_{29}$.
- $\delta'_{out} = (\text{ARC} \circ \text{MB} \circ \text{Sh})^{-1}(\delta_{out})$.
- $R_i = \{(Y, Y \oplus \delta'_{out}), Y \in \mathbb{F}_2^{8 \cdot 8}\}$.



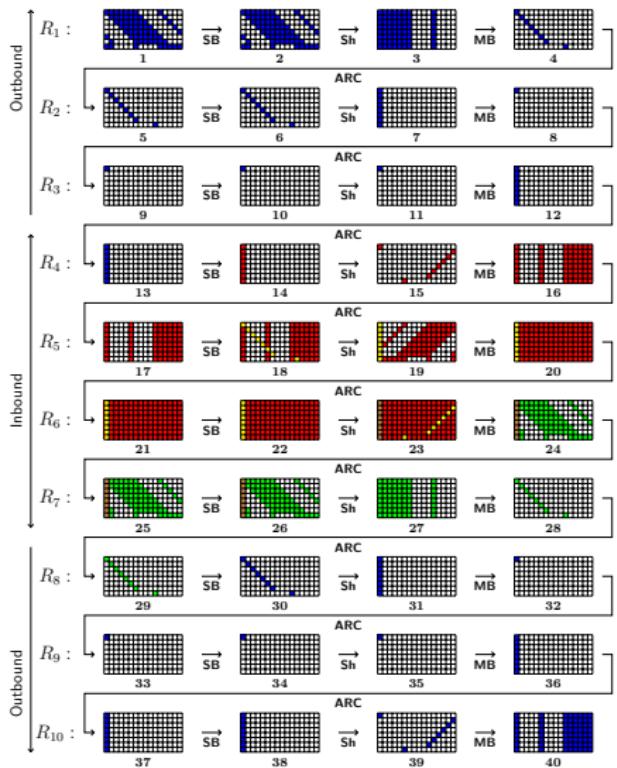
Deterministic propagation of lists R_i

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- $\delta'_{in} = \text{ARC} \circ \text{MB} \circ \text{Sh}(\delta_{in})$.
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- Choose $\delta_{out} \in P_{29}$.
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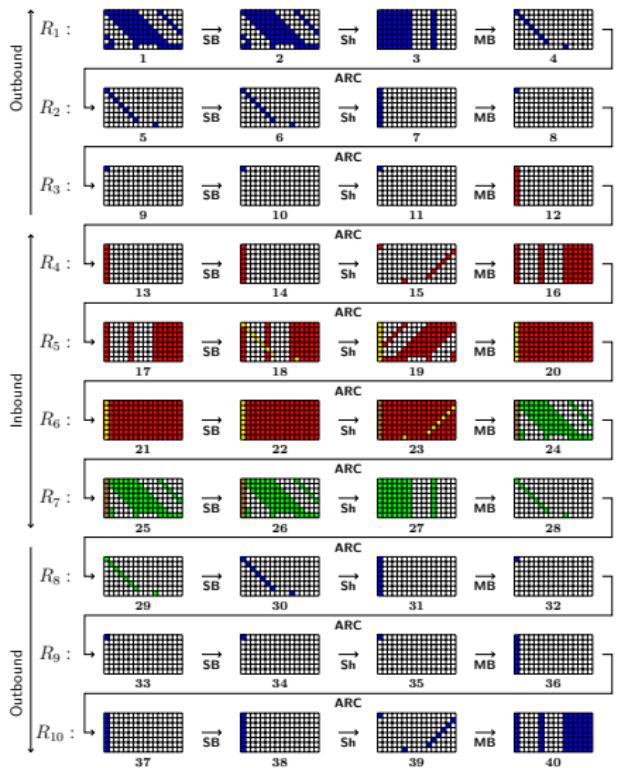
Merging lists

- Choose $\delta_{in} \in P_{14}$.
- $\delta'_{in} = \text{ARC} \circ \text{MB} \circ \text{Sh}(\delta_{in})$.
- $L_i = \{(X, X \oplus \delta'_{in}), X \in \mathbb{F}_2^{8 \cdot 8}\}$.
- $L'_i = \text{Sh} \circ \text{SB} \circ \text{R}_5(L_i)$.
- Choose $\delta_{out} \in P_{29}$.
- $\delta'_{out} = (\text{ARC} \circ \text{MB} \circ \text{Sh})^{-1}(\delta_{out})$.
- $R_i = \{(Y, Y \oplus \delta'_{out}), Y \in \mathbb{F}_2^{8 \cdot 8}\}$.
- $R'_i = (\text{SB} \circ \text{ARC} \circ \text{MB})^{-1}(R_i)$.
- Merging lists L'_i and R'_i .
 (Guess and Determine)
 We find a match with $\mathcal{C} \simeq 2^{280}$ and
 $\mathcal{M} \simeq 2^{64}$.



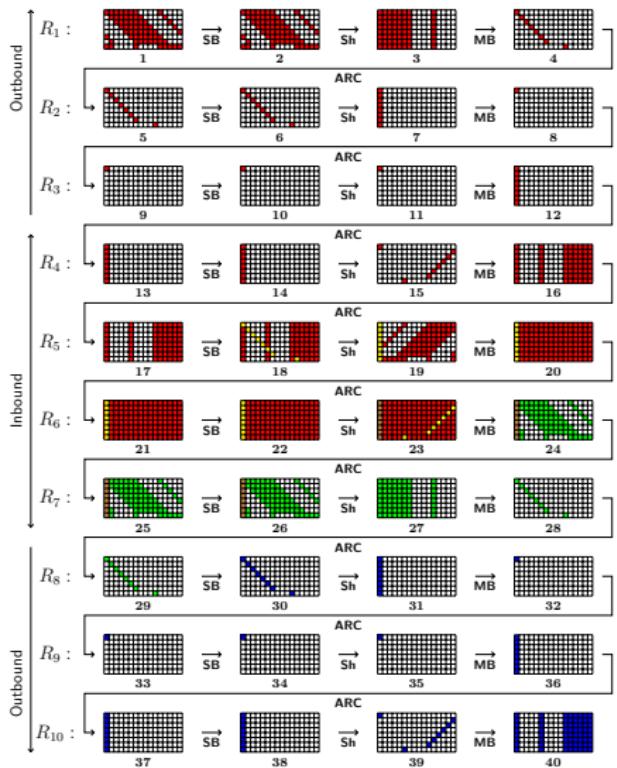
Probabilistic transition through MB⁻¹

- Choose $\delta_{in} \in P_{14}$.
- $\delta'_{in} = \text{ARC} \circ \text{MB} \circ \text{Sh}(\delta_{in})$.
- $L_i = \{(X, X \oplus \delta'_{in}), X \in \mathbb{F}_2^{8 \cdot 8}\}$.
- $L'_i = \text{Sh} \circ \text{SB} \circ \text{R}_5(L_i)$.
- Choose $\delta_{out} \in P_{29}$.
- $\delta'_{out} = (\text{ARC} \circ \text{MB} \circ \text{Sh})^{-1}(\delta_{out})$.
- $R_i = \{(Y, Y \oplus \delta'_{out}), Y \in \mathbb{F}_2^{8 \cdot 8}\}$.
- $R'_i = (\text{SB} \circ \text{ARC} \circ \text{MB})^{-1}(R_i)$.
- Merging lists L'_i and R'_i .
 (Guess and Determine)
- $\mathbb{P}(P_{12} \rightarrow P_{11}) = 2^{-7.8}$.



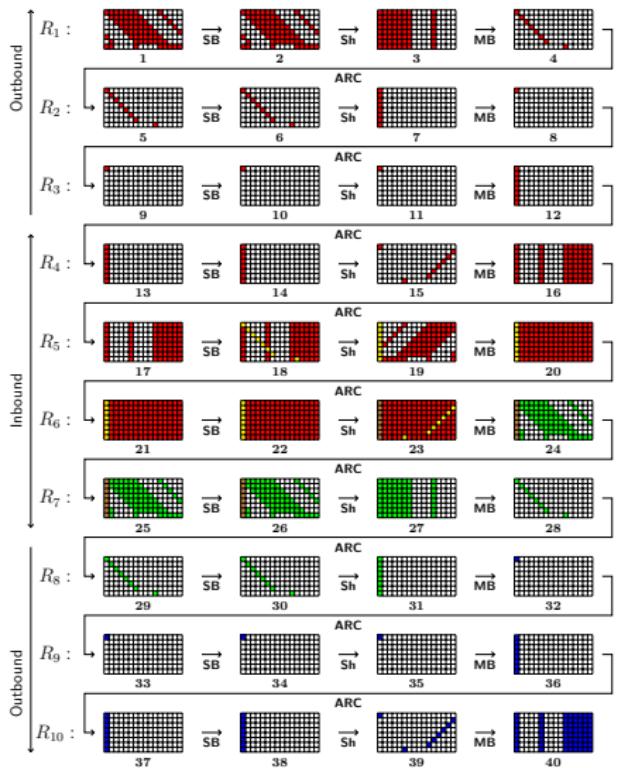
Deterministic transition

- Choose $\delta_{in} \in P_{14}$.
- $\delta'_{in} = \text{ARC} \circ \text{MB} \circ \text{Sh}(\delta_{in})$.
- $L_i = \{(X, X \oplus \delta'_{in}), X \in \mathbb{F}_2^{8 \cdot 8}\}$.
- $L'_i = \text{Sh} \circ \text{SB} \circ \text{R}_5(L_i)$.
- Choose $\delta_{out} \in P_{29}$.
- $\delta'_{out} = (\text{ARC} \circ \text{MB} \circ \text{Sh})^{-1}(\delta_{out})$.
- $R_i = \{(Y, Y \oplus \delta'_{out}), Y \in \mathbb{F}_2^{8 \cdot 8}\}$.
- $R'_i = (\text{SB} \circ \text{ARC} \circ \text{MB})^{-1}(R_i)$.
- Merging lists L'_i and R'_i .
 (Guess and Determine)
- $\mathbb{P}(P_{12} \rightarrow P_{11}) = 2^{-7 \cdot 8}$.



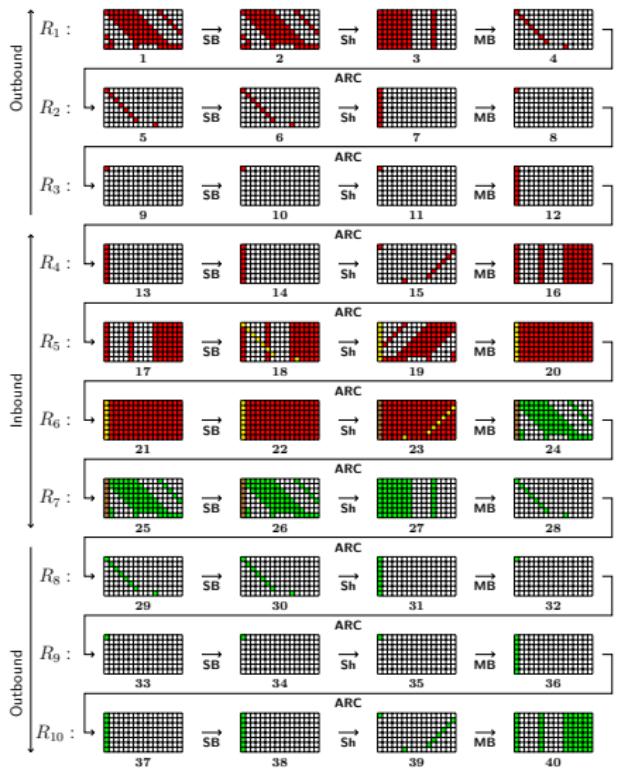
Probabilistic transition through MB

- Choose $\delta_{in} \in P_{14}$.
- $\delta'_{in} = \text{ARC} \circ \text{MB} \circ \text{Sh}(\delta_{in})$.
- $L_i = \{(X, X \oplus \delta'_{in}), X \in \mathbb{F}_2^{8 \cdot 8}\}$.
- $L'_i = \text{Sh} \circ \text{SB} \circ \text{R}_5(L_i)$.
- Choose $\delta_{out} \in P_{29}$.
- $\delta'_{out} = (\text{ARC} \circ \text{MB} \circ \text{Sh})^{-1}(\delta_{out})$.
- $R_i = \{(Y, Y \oplus \delta'_{out}), Y \in \mathbb{F}_2^{8 \cdot 8}\}$.
- $R'_i = (\text{SB} \circ \text{ARC} \circ \text{MB})^{-1}(R_i)$.
- Merging lists L'_i and R'_i .
 (Guess and Determine)
- $\mathbb{P}(P_{12} \rightarrow P_{11}) = 2^{-7 \cdot 8}$.
- $\mathbb{P}(P_{31} \rightarrow P_{32}) = 2^{-7 \cdot 8}$.



Deterministic transition

- Choose $\delta_{in} \in P_{14}$.
- $\delta'_{in} = \text{ARC} \circ \text{MB} \circ \text{Sh}(\delta_{in})$.
- $L_i = \{(X, X \oplus \delta'_{in}), X \in \mathbb{F}_2^{8 \cdot 8}\}$.
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- Choose $\delta_{out} \in P_{29}$.
- $\delta'_{out} = (\text{ARC} \circ \text{MB} \circ \text{Sh})^{-1}(\delta_{out})$.
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- Merging lists L'_i and R'_i .
 (Guess and Determine)
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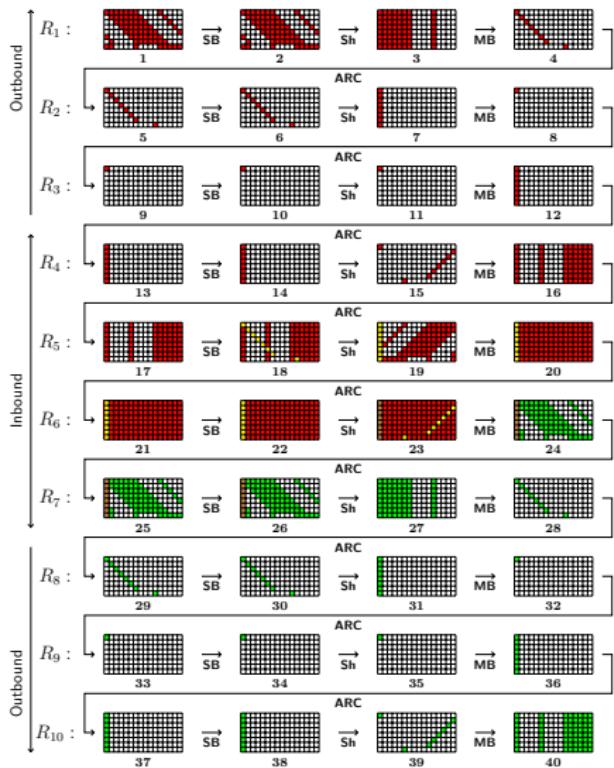


10-round distinguisher

- Choose $\delta_{in} \in P_{14}$.
- $\delta'_{in} = \text{ARC} \circ \text{MB} \circ \text{Sh}(\delta_{in})$.
- $L_i = \{(X, X \oplus \delta'_{in}), X \in \mathbb{F}_2^{8 \cdot 8}\}$.
- $L'_i = \text{Sh} \circ \text{SB} \circ \text{R}_5(L_i)$.
- Choose $\delta_{out} \in P_{29}$.
- $\delta'_{out} = (\text{ARC} \circ \text{MB} \circ \text{Sh})^{-1}(\delta_{out})$.
- $R_i = \{(Y, Y \oplus \delta'_{out}), Y \in \mathbb{F}_2^{8 \cdot 8}\}$.
- $R'_i = (\text{SB} \circ \text{ARC} \circ \text{MB})^{-1}(R_i)$.
- Merging lists L'_i and R'_i .
 (Guess and Determine)
 - $\mathbb{P}(P_{12} \rightarrow P_{11}) = 2^{-7 \cdot 8}$.
 - $\mathbb{P}(P_{31} \rightarrow P_{32}) = 2^{-7 \cdot 8}$.
- Overall complexity:

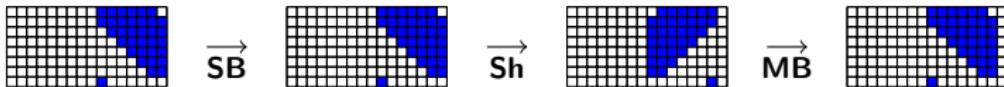
$$\begin{cases} \mathcal{C} & \simeq 2^{112} \cdot 2^{280} = 2^{392} < 2^{448} \\ \mathcal{M} & \simeq 2^{7 \cdot 8} \end{cases}$$

Distinguisher!



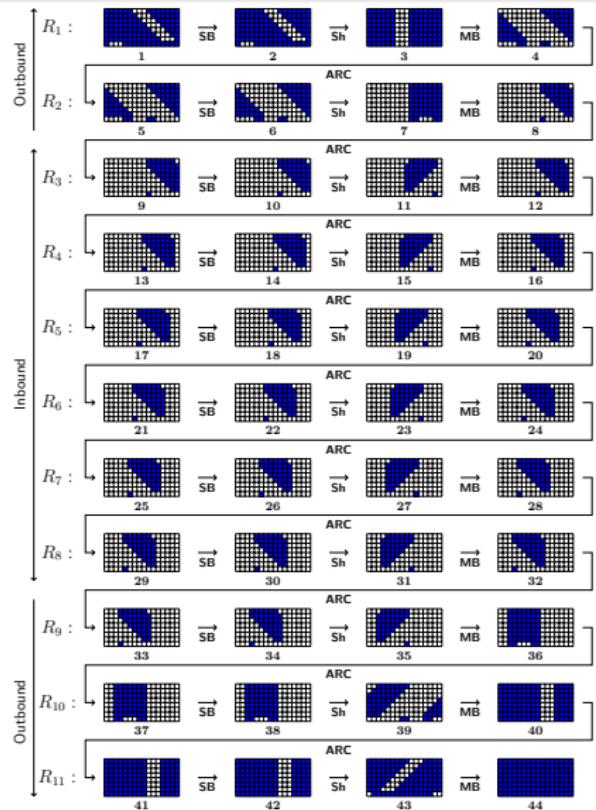
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- 2 10-round Rebound Attack on Grøstl₅₁₂ Permutations
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Mixed-Integer Linear Programming



Probabilistic step through MB of probability $2^{-22.8}$.

11-round truncated differential path

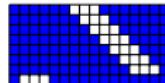


Re-Rebound Attack

- **Inbound phase:** Collect many samples designed to satisfy **6 middle rounds** of the truncated differential path. Find couples of state values compatible with **3 differential values** δ_1 , δ_2 and δ_3 propagated forward and backward.
- **Outbound phase:** Find among those couples of state values one satisfying both probabilistic transitions towards the first and last rounds.

Generic limited-birthday algorithm complexity

- Initial state:



$$\dim(E_{in}) = 104 \cdot 8$$

- Final state:



$$\dim(E_{out}) = 104 \cdot 8$$

- Computational complexity:

$$\log_2(\mathcal{C}_{gen}) = \frac{128 - 104}{2} \cdot 8 = 12 \cdot 8 = 96$$

Plausibility

- Sequence of numbers of active bytes:

$104 \xrightarrow{R_1} 53 \xrightarrow{R_2} 34 \xrightarrow{R_3} 34 \xrightarrow{R_4} 34 \xrightarrow{R_5} 34 \xrightarrow{R_6} 34 \xrightarrow{R_7} 34 \xrightarrow{R_8} 34 \xrightarrow{R_9} 53 \xrightarrow{R_{10}} 104 \xrightarrow{R_{11}} 128$

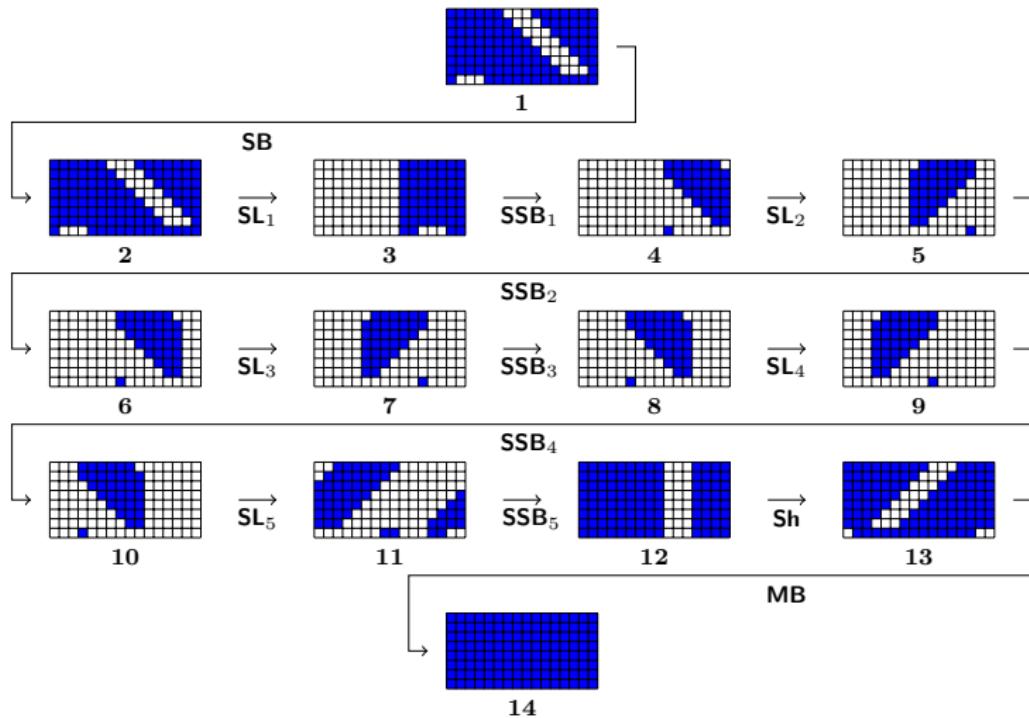
- $2^{(104+128)\cdot 8}$ possible initial states.

- Probabilistic transitions :

- 1 transition with probability $2^{-51\cdot 8}$
- 7 transitions with probability $2^{-22\cdot 8}$
- 1 transitions with probability $2^{-3\cdot 8}$

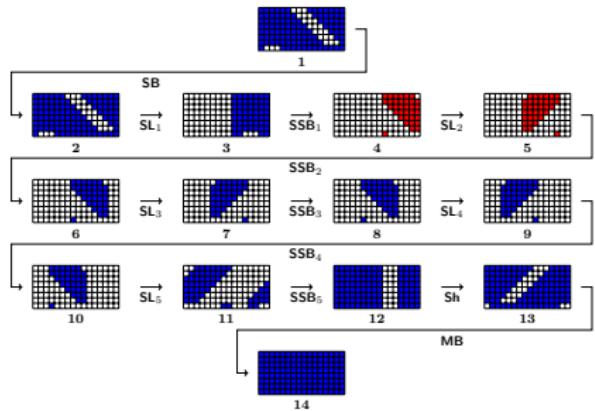
$\Rightarrow 2^{24\cdot 8}$ such differences are expected.

Super SBOX description



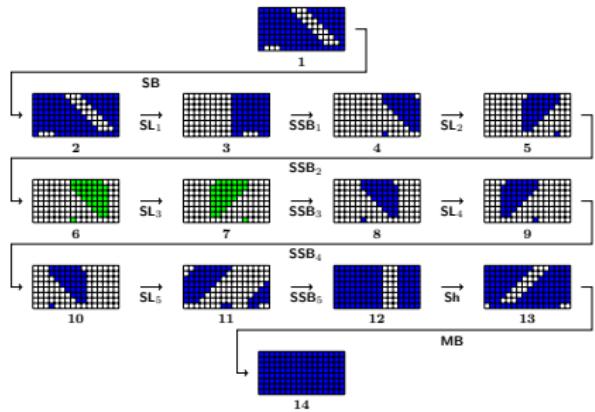
Computation of differential set Δ_1

$$\Delta_1 = \{\delta_1 \in P_4 \mid \delta'_1 = \mathbf{SL}_2(\delta_1) \in P_5\}.$$



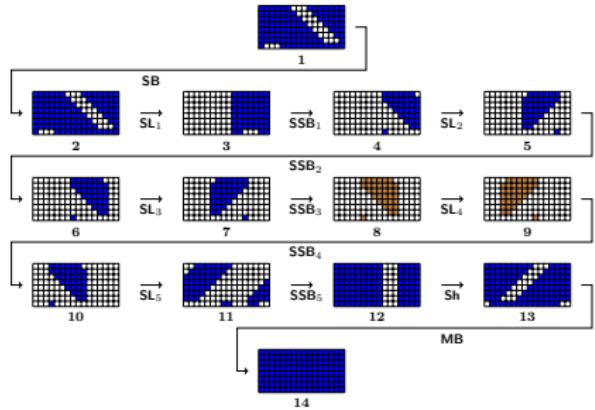
Computation of differential set Δ_2

$$\Delta_2 = \{\delta_2 \in P_6 \mid \delta'_2 = \mathbf{SL}_3(\delta_2) \in P_7\}.$$



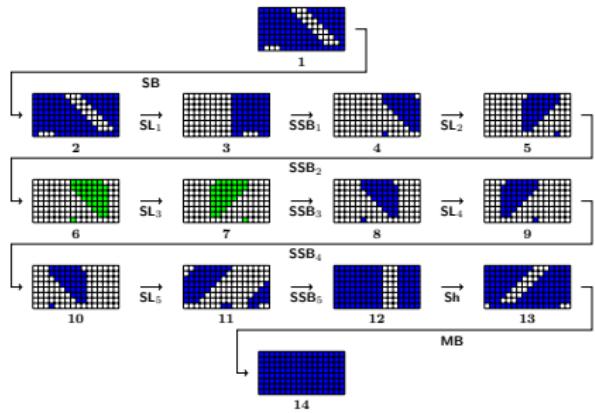
Computation of differential set Δ_3

$$\Delta_3 = \{\delta_3 \in P_8 \mid \delta'_3 = \mathbf{SL}_4(\delta_3) \in P_9\}.$$



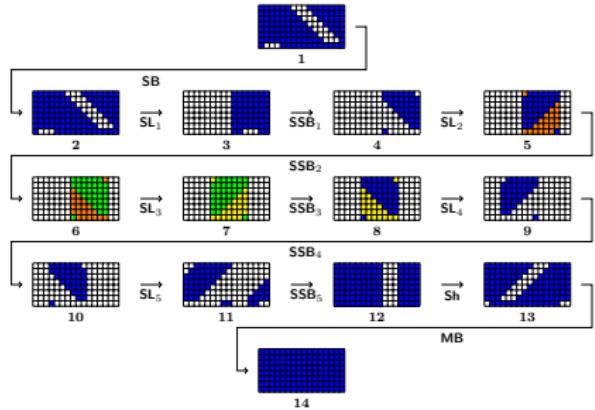
Selection of a differential δ_2

- Choose $\delta_2 \in \Delta_2$.



Computation of 7 lists C_i and 7 lists C'_i

- Choose $\delta_2 \in \Delta_2$.
- Compute C_i and C'_i :

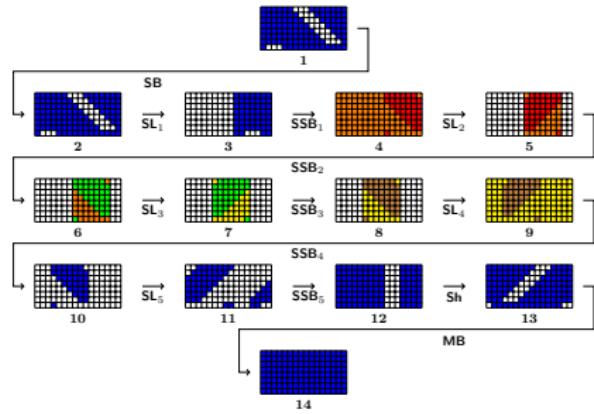


Column by column, complexity: $\mathcal{C} \simeq 2^{7 \cdot 8}$, $\mathcal{M} \simeq 2^{7 \cdot 8}$

$$\begin{aligned} C_i &= \{ (X, Y = X \oplus (\delta_2)_{|i}) \mid \mathbf{SSB}_2^{-1}(X) \oplus \mathbf{SSB}_2^{-1}(Y) \in (P_5)_{|i} \}, \\ C'_i &= \{ (X, Y = X \oplus (\delta'_2)_{|j}) \mid \mathbf{SSB}_3(X) \oplus \mathbf{SSB}_3(Y) \in (P_8)_{|j} \}. \end{aligned}$$

Computation of lists E and F

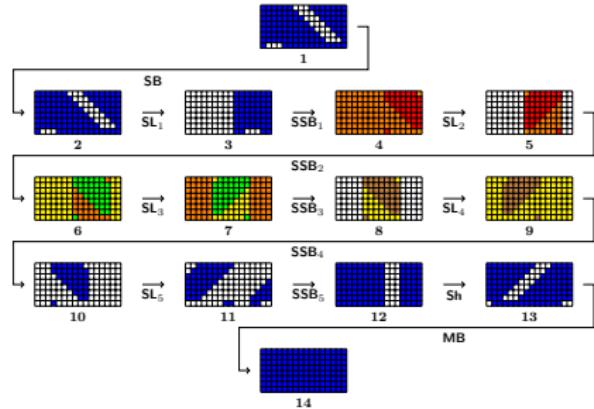
- Choose $\delta_2 \in \Delta_2$.
- Compute C_i and C'_i .
- Compute E and F :



To construct $|E| = 2^{6 \cdot 8}$ and $|F| = 2^{6 \cdot 8}$, we need $\mathcal{C}_3 \simeq 2^{6 \cdot 8}$ and $\mathcal{M}_3 \simeq 2^{6 \cdot 8}$.

Merging lists

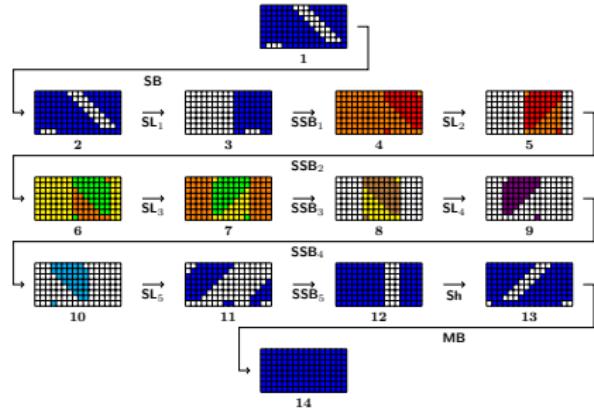
- Choose $\delta_2 \in \Delta_2$.
- Compute C_i and C'_i .
- Compute E and F .
- Merging E and F .
 (1st Guess and Determine)



$\mathbb{P}(e \in E \text{ and } f \in F \text{ admits a matching completion}) = 2^{-12.8}$
 Any fitting choice admits $2^{28.8}$ matching completions
 We find such choice with $\mathcal{C}_4 \simeq 2^{7.8}$ and $\mathcal{M}_4 \simeq 2^{6.8}$

Tricky choice

- Choose $\delta_2 \in \Delta_2$.
- Compute C_i and C'_i .
- Compute E and F .
- Merging E and F .
 (1st Guess and Determine)
- Choose $(s, s \oplus \delta_3)$.
 (2nd Guess and Determine)



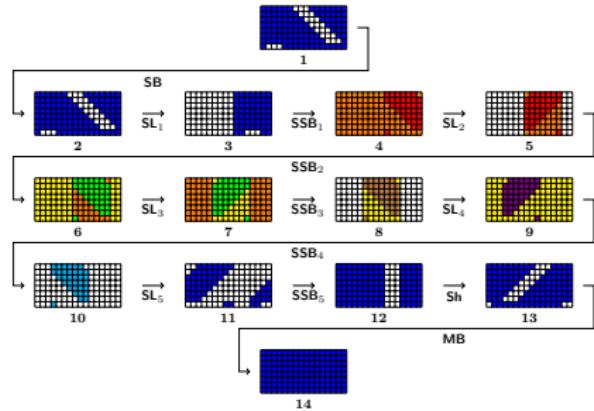
$$\mathbb{P}((s, s \oplus \delta_3) \text{ admits a completion}) = 2^{-12 \cdot 8}$$

Any fitting choice admits $2^{72 \cdot 8}$ matching completions

We find such a choice with $\mathcal{C}_5 \simeq 2^{3 \cdot 8}$ and $\mathcal{M}_5 \simeq 2^{3 \cdot 8}$

Merging completions

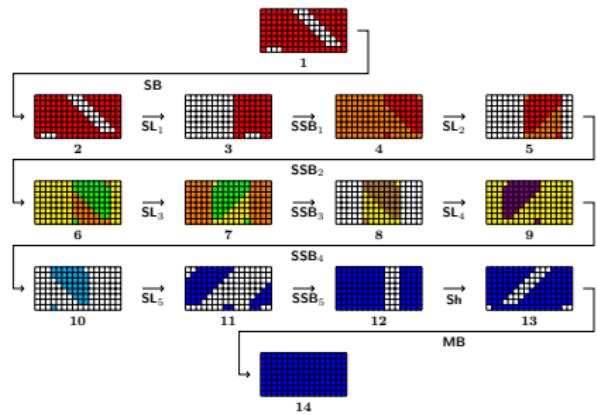
- Choose $\delta_2 \in \Delta_2$.
- Compute C_i and C'_i .
- Compute E and F .
- Merging E and F .
 (1st Guess and Determine)
- Choose $(s, s \oplus \delta_3)$.
 (2nd Guess and Determine)
- Merging completions.
 (3rd Guess and Determine)



$2^{6 \cdot 8}$ complete state values are in the intersection of both completions
 We compute and store them with $\mathcal{C}_6 \simeq 2^{9 \cdot 8}$ and $\mathcal{M}_6 \simeq 2^{7 \cdot 8}$

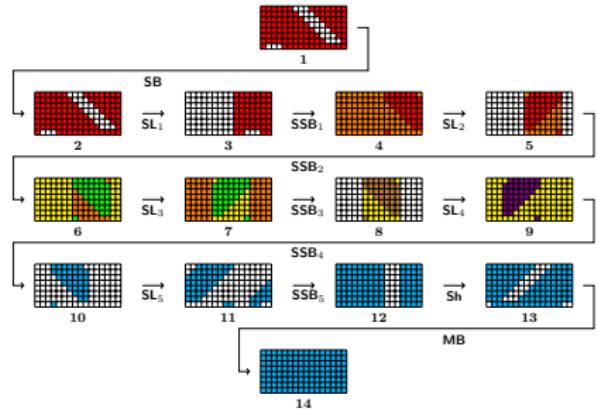
Probabilistic transition through SSB₁⁻¹

- Choose $\delta_2 \in \Delta_2$.
- Compute C_i and C'_i .
- Compute E and F .
- Merging E and F .
 (1st Guess and Determine)
- Choose $(s, s \oplus \delta_3)$.
 (2nd Guess and Determine)
- Merging completions.
 (3rd Guess and Determine)
- $\mathbb{P}(P_4 \rightarrow P_3) = 2^{-3 \cdot 8}$.



Probabilistic transition through SL₅

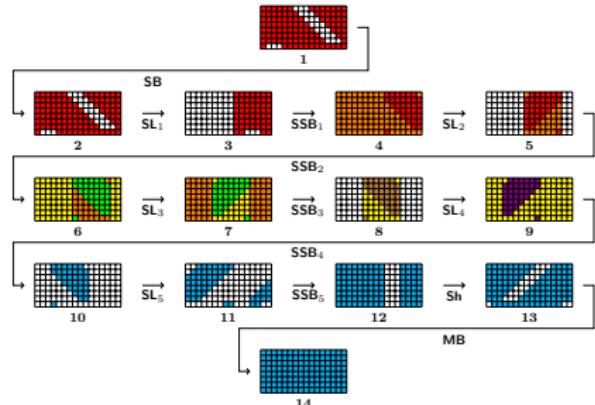
- Choose $\delta_2 \in \Delta_2$.
- Compute C_i and C'_i .
- Compute E and F .
- Merging E and F .
 (1st Guess and Determine)
- Choose $(s, s \oplus \delta_3)$.
 (2nd Guess and Determine)
- Merging completions.
 (3rd Guess and Determine)
- $\mathbb{P}(P_4 \rightarrow P_3) = 2^{-3 \cdot 8}$.
- $\mathbb{P}(P_{10} \rightarrow P_{11}) = 2^{-3 \cdot 8}$.



11-round distinguisher

- Choose $\delta_2 \in \Delta_2$.
- Compute C_i and C'_i .
- Compute E and F .
- Merging E and F .
 (1st Guess and Determine)
- Choose $(s, s \oplus \delta_3)$.
 (2nd Guess and Determine)
- Merging completions.
 (3rd Guess and Determine)
- $\mathbb{P}(P_4 \rightarrow P_3) = 2^{-3 \cdot 8}$.
- $\mathbb{P}(P_{10} \rightarrow P_{11}) = 2^{-3 \cdot 8}$.
- Overall complexity:

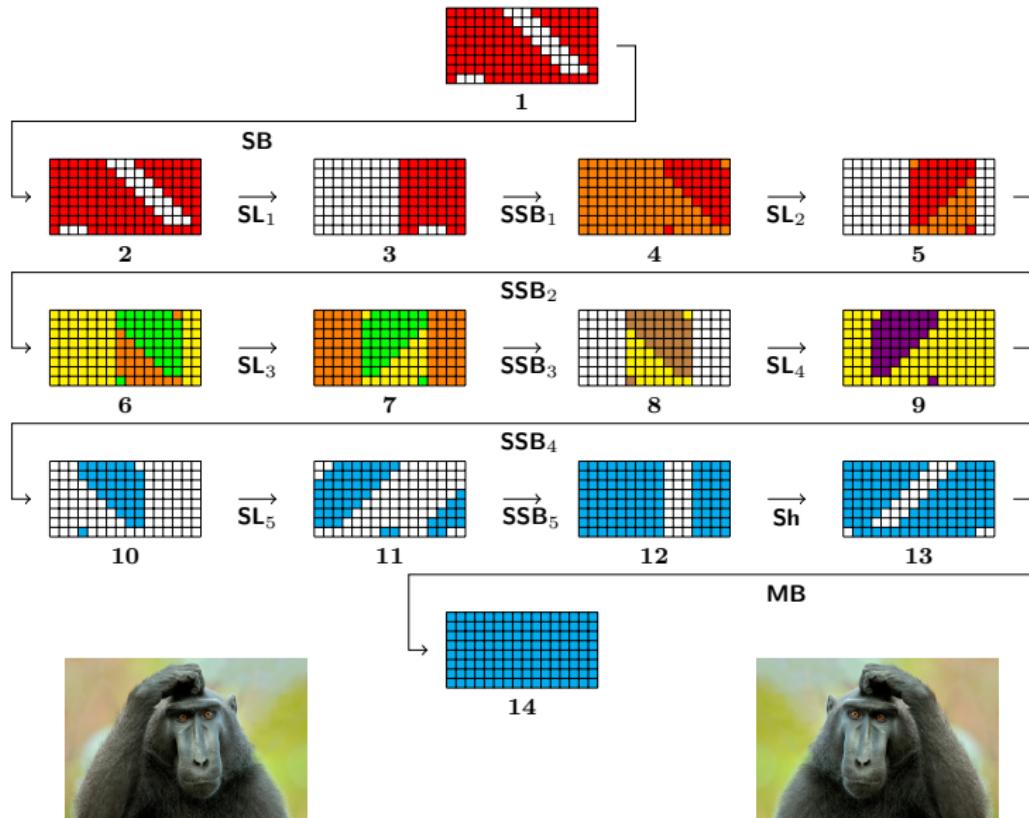
$$\begin{cases} \mathcal{C} & \simeq 2^{9 \cdot 8} < 2^{96} \\ \mathcal{M} & \simeq 2^{7 \cdot 8} \end{cases}$$



Distinguisher!

- First rebound attack on 11 round of Grøstl₅₁₂'s permutations.
- 12-round truncated differential path is statistically realized.
- It seems difficult to derive a distinguisher for 12 rounds.
- These methods shall generalize to all AES-like permutations.

Questions?



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