Radboud University

Sound Hashing Modes of Arbitrary Functions, Permutations, and Block Ciphers (SoK)

Joan Daemen[1] Bart Mennink[1] Gilles Van Assche[2]

Fast Software Encryption

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[1]Radboud University
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Hash function example 1: SHA-256

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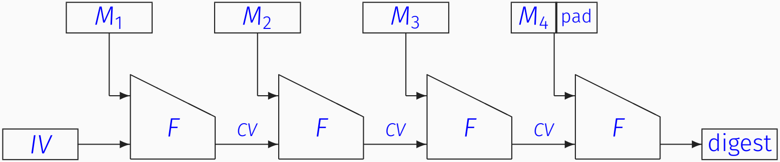
Hash function h from compression function F with **Merkle-Damgård**:



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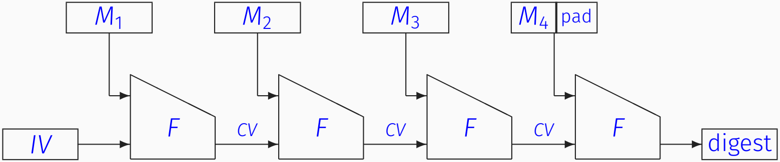

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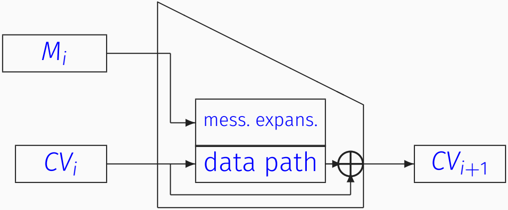

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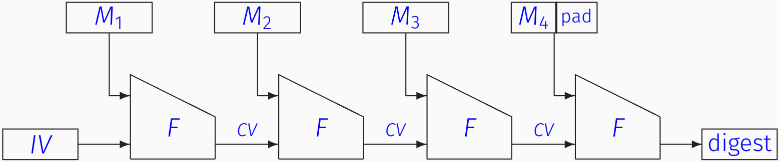

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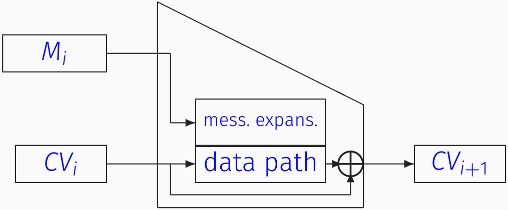
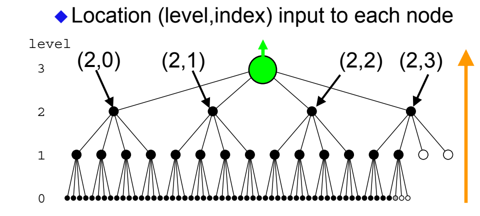


Underlying primitive: block cipher with 256-bit block and 512-bit *key*

Example 2: MD6 [Rivest et al. 2008]

Hash function h from CF F with dedicated tree hash mode:



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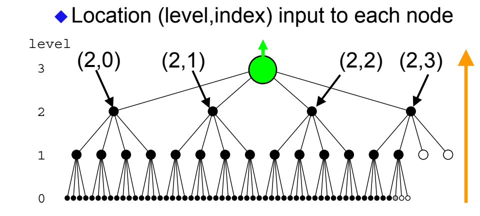

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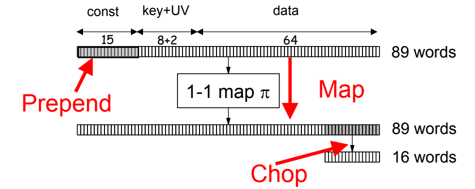

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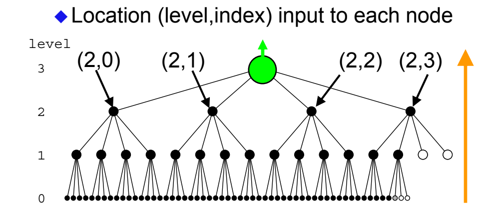

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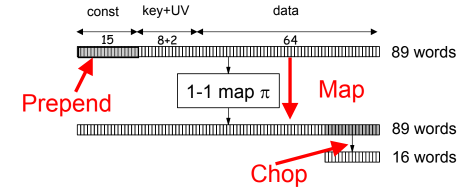

Underlying primitive: 5696-bit permutation

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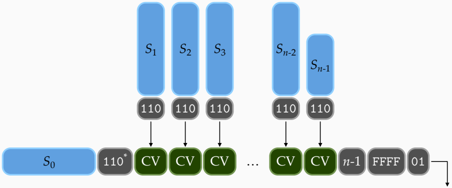

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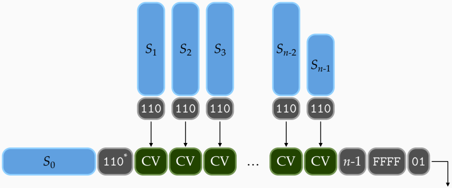

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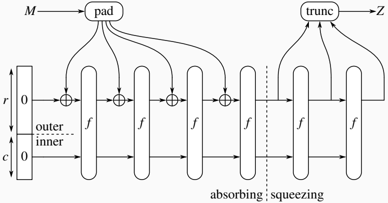

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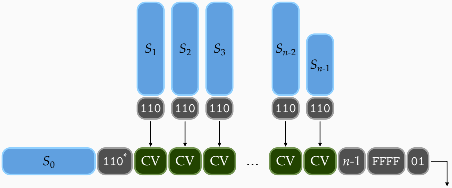

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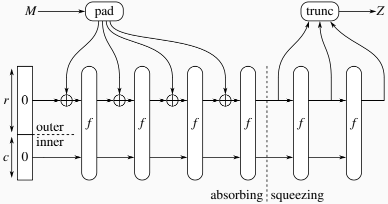

Underlying primitive: 1600-bit permutation KECCAK-p[12]

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- In other words, they bound the success probability of generic attacks

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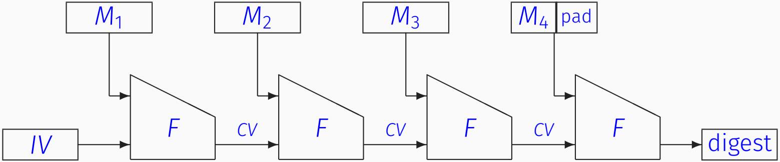

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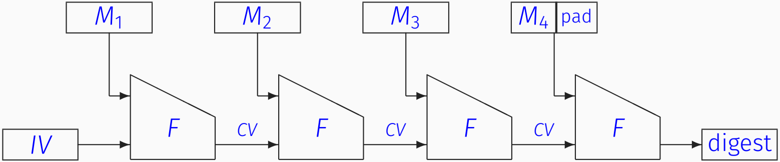

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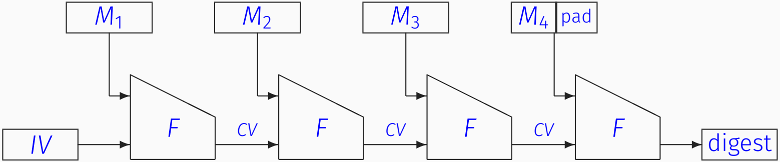

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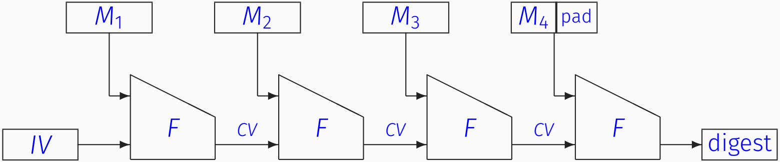

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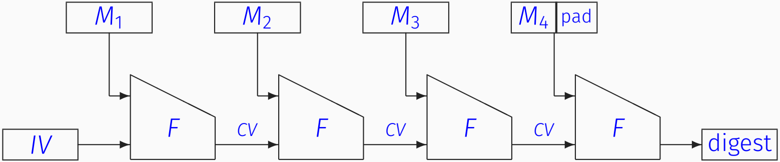

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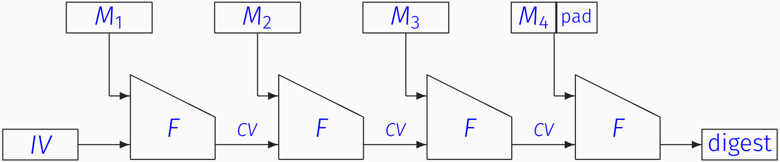

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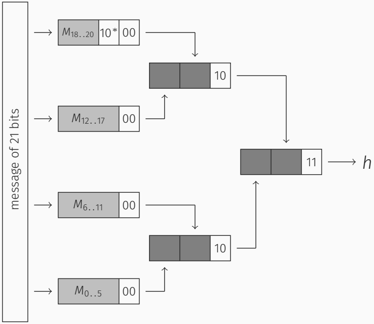
- Affect all old-style hash standards: MD5, SHA-1 and all SHA-2

Hashing, scope of this SoK paper



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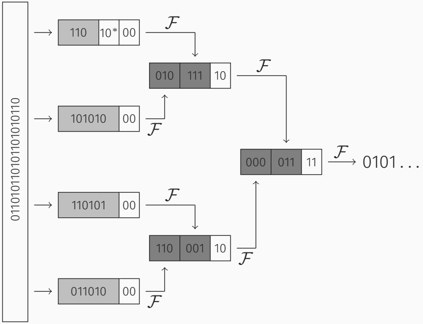

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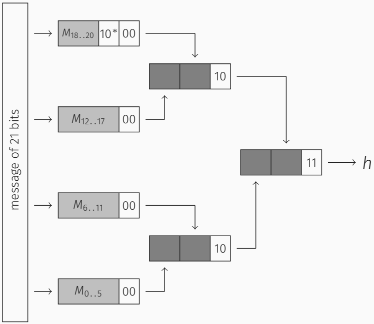

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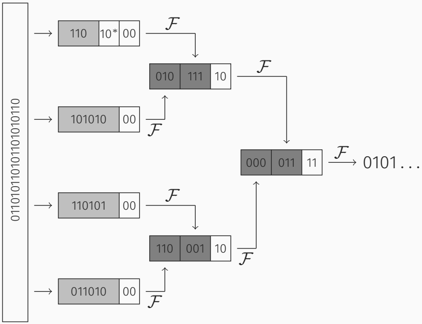

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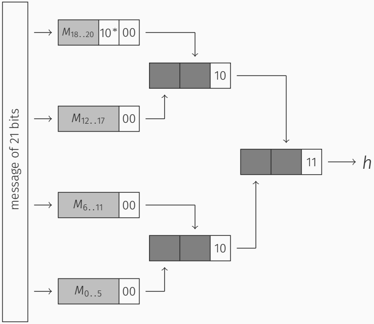

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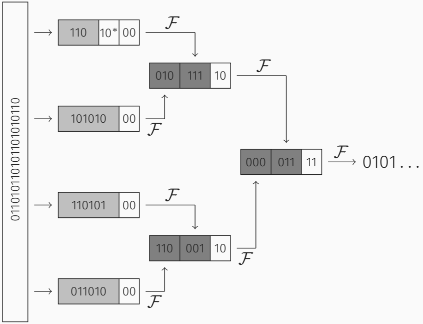

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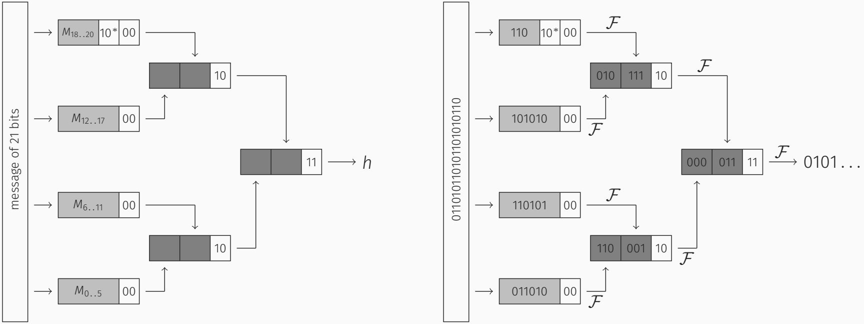

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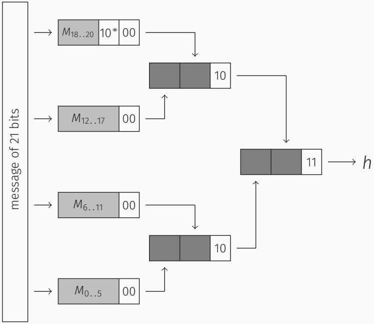

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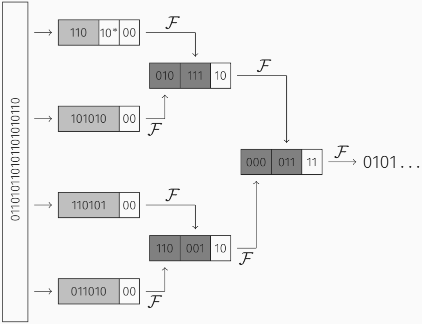

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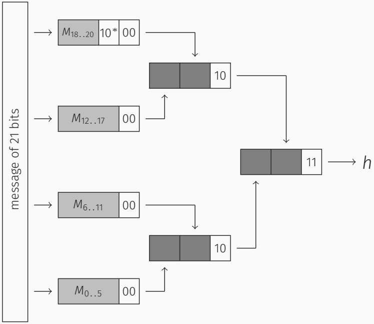

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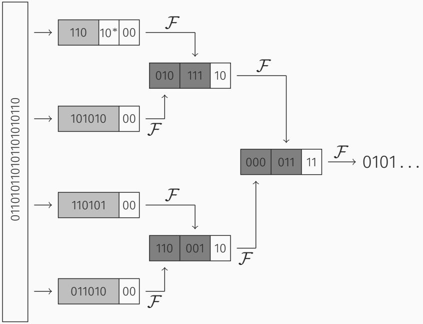

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  - (truncated) block cipher

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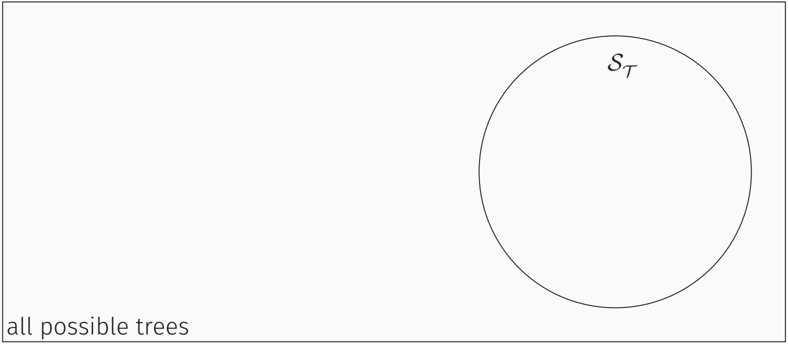
We prove it is hard to distinguish $\mathcal{H}$ from $\mathcal{RO}$ if $\mathcal{T}$ satisfies certain conditions:

- For all cases:
  - message-decodability
  - subtree-freeness
  - radical-decodability
- For permutations and block ciphers:
  - leaf-anchoring

Trees and the set $\mathcal{S}_{\mathcal{T}}$



$\mathcal{S}_{\mathcal{T}}$: the set of all possible trees that can be generated by mode $\mathcal{T}$

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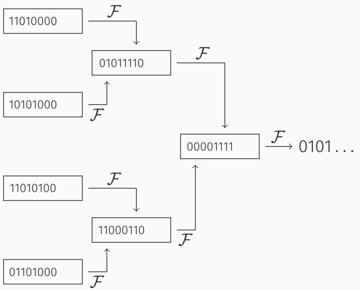
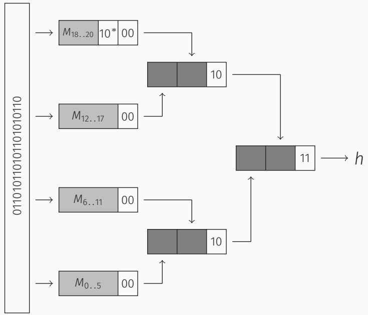


$\Longrightarrow$



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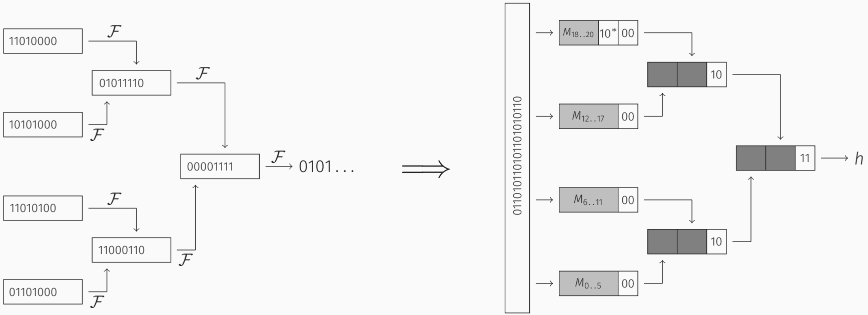

$\forall S \in \mathcal{S}_{\mathcal{T}}$ there exists an algorithm for decoding S to (M, Z)

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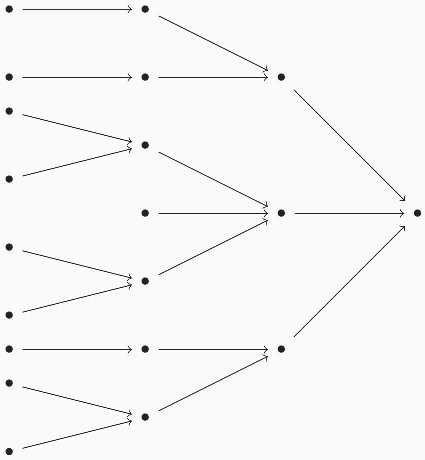

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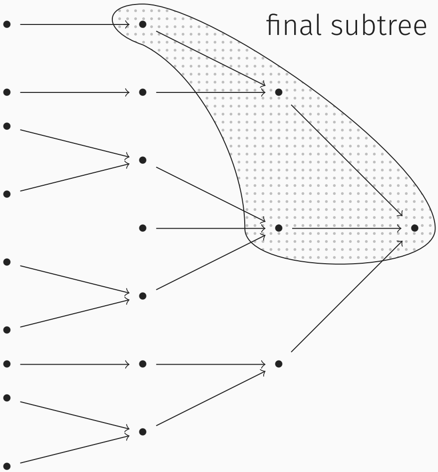

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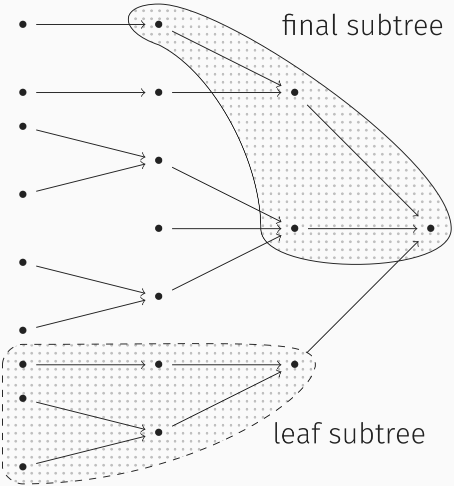

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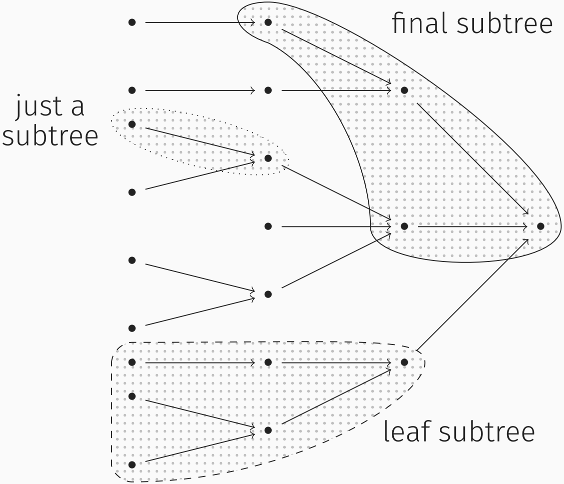

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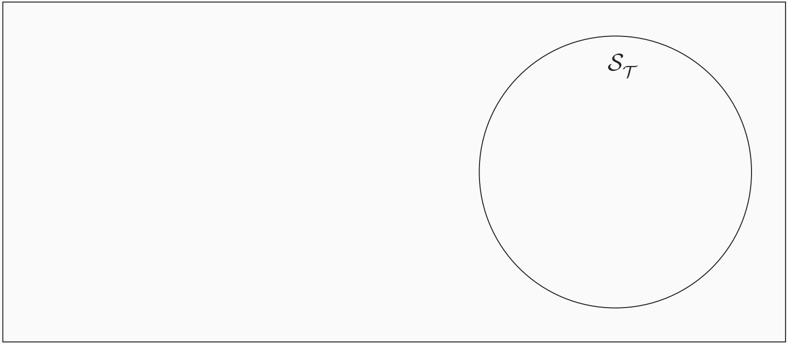

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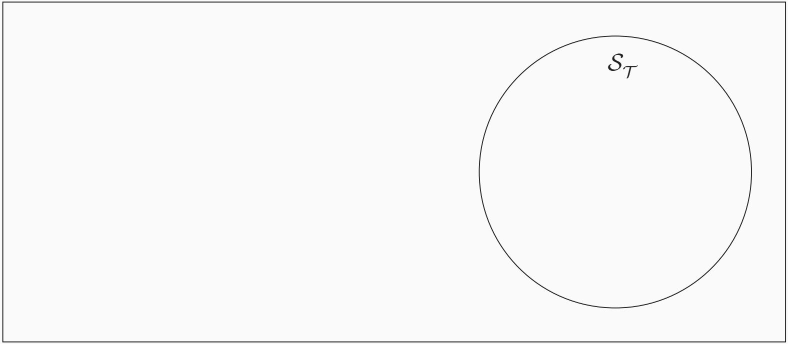

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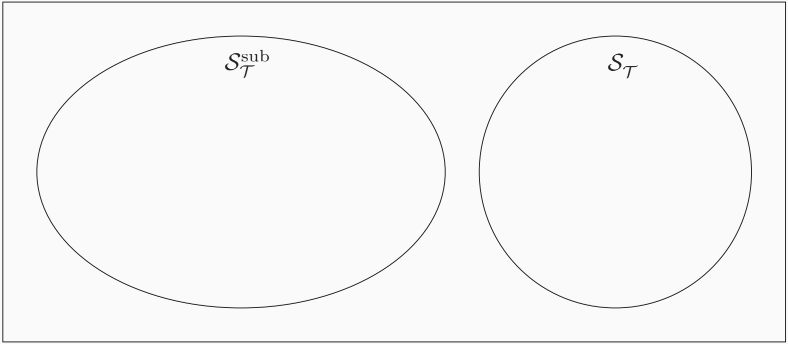

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Subtree-freeness: $\mathcal{S}_{\mathcal{T}} \cap \mathcal{S}_{\mathcal{T}}^{\mathrm{sub}} = \emptyset$

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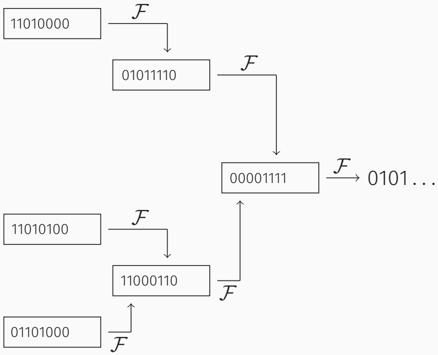

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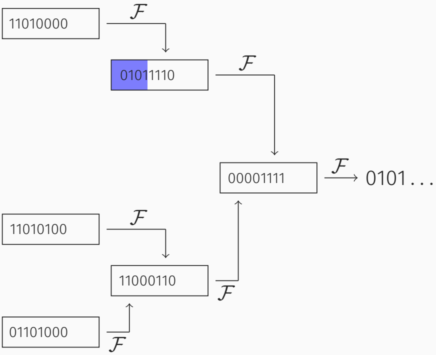

Radical: a CV that has no $\mathcal{F}$-pre-image

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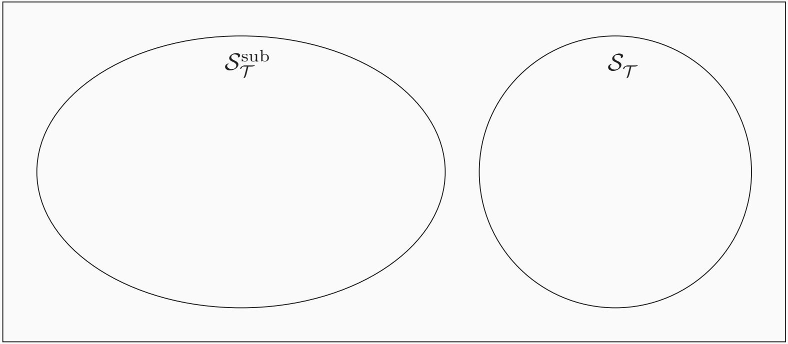

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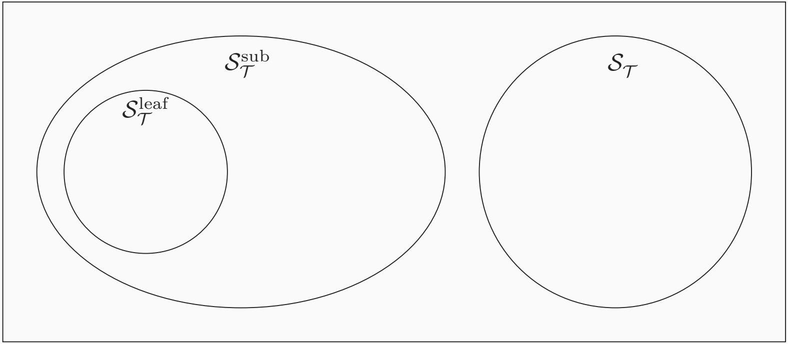

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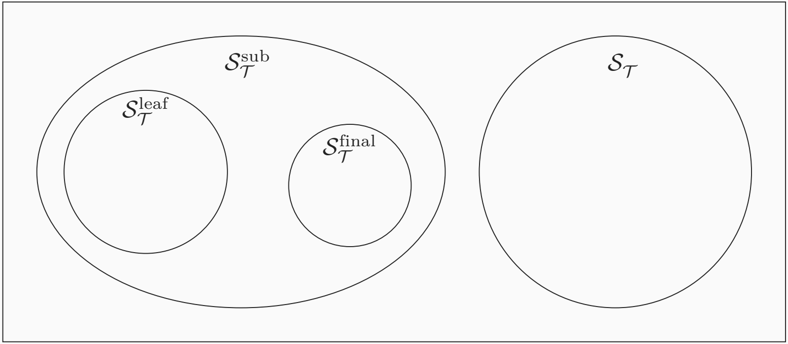

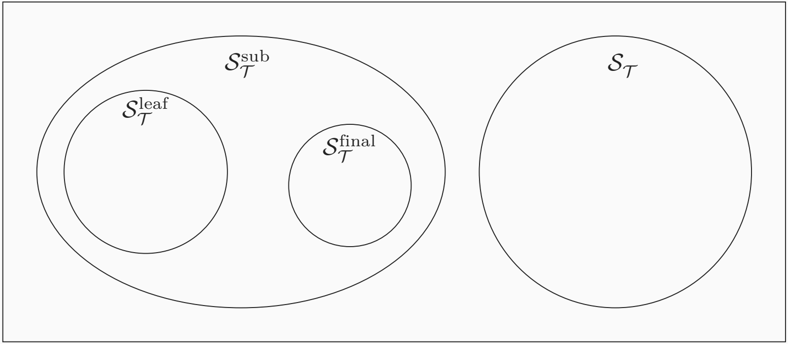

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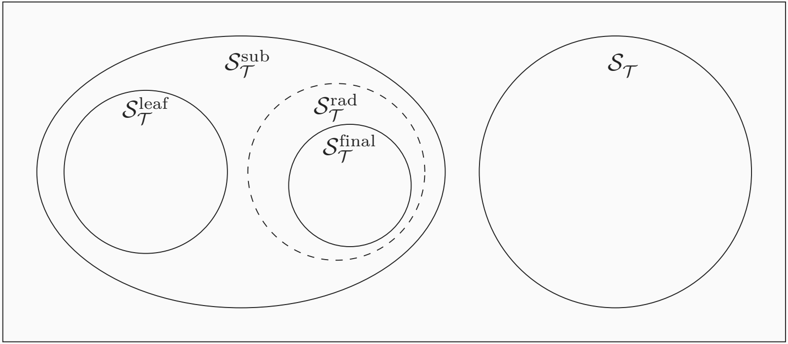

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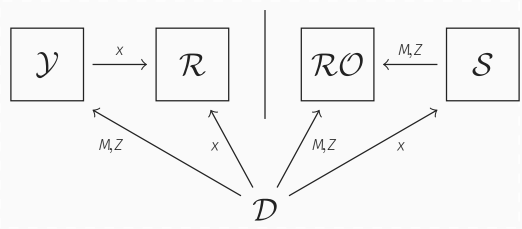
Radical-decodability, actually: this is true for all subtrees in some set $\mathcal{S}_{\mathcal{T}}^{\text{rad}}$ that includes $\mathcal{S}_{\mathcal{T}}^{\text{final}}$

Adversary model: differentiating from a random oracle



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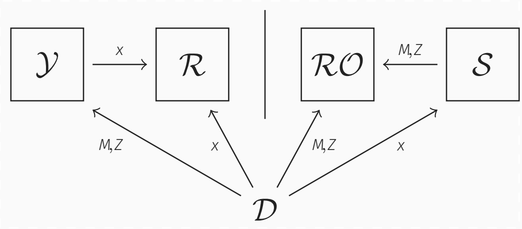

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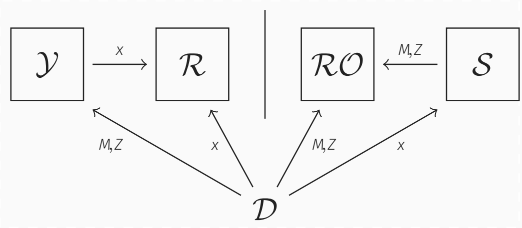

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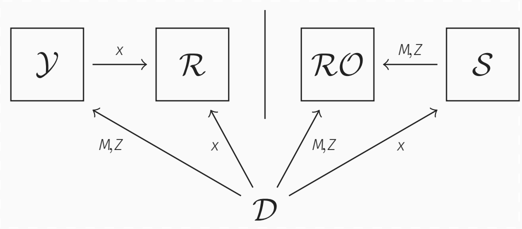
Adversary model: differentiating from a random oracle



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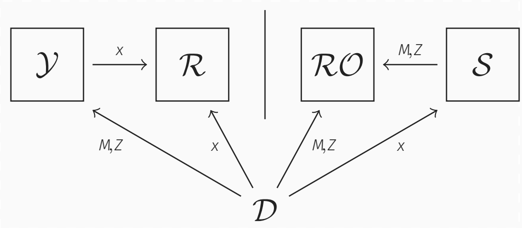

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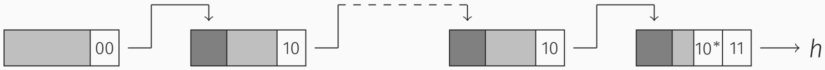
- Adding a feedforward à la Davies-Meyer does **not** help

With a compression function:



Minimum solutions for sequential hashing

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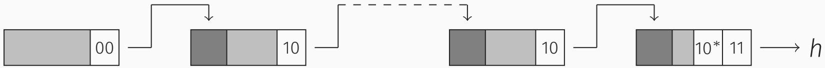
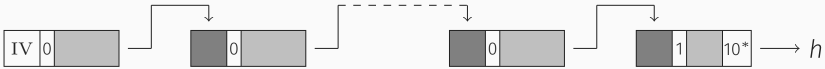


With a truncated permutation or block cipher:



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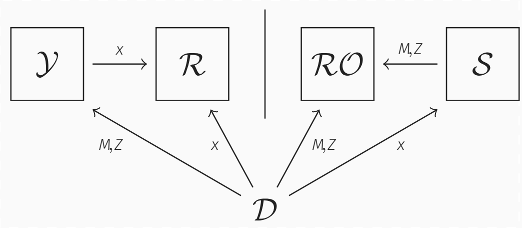
Thanks for your attention!

Intuition: why this works



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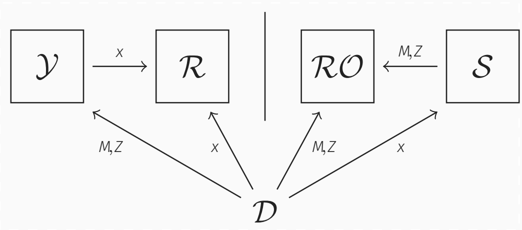

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- Things break down when CVs collide

An example that is not radical-decodable