Generalized Nonlinear Invariant Attack and a New Design Criterion for Round Constants

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Presented by René Rodríguez[3]

Fast Software Encryption 2019, Paris, France

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March 25th, 2019

Summary of the talk

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Why is it important?

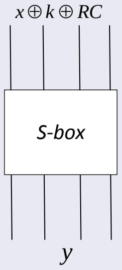
Commonly induce distinguishing attacks, especially lightweight block ciphers are susceptible to this kind of cryptanalysis.

Example

Let $g : F_2^4 \to F_2$ *be a nonlinear function defined as*

$$g(a_4, a_3, a_2, a_1) = a_4 a_3 \oplus a_3 \oplus a_2 \oplus a_1$$

$$a_i = x_i \oplus k_i \oplus RC_i$$



If $k_3 \oplus RC_3 = 0$ *and* $k_4 \oplus RC_4 = 0$,
then, $g(x_4, x_3, x_2, x_1) \oplus g(y_4, y_3, y_2, y_1) = c$ *for all* x

If $k_3 \oplus RC_3 \neq 0$ *or* $k_4 \oplus RC_4 \neq 0$, *then,*
$g(x_4, x_3, x_2, x_1) \oplus g(y_4, y_3, y_2, y_1) \neq c$ *for all* x

Vulnerable lightweight block ciphers

- PRINT-cipher [Leander et al. 2011]
- iSCREAM, Robin, Zorro [Leander, Minaud, Rønjom 2015]
- Midori-64 [Guo et al. 2016]
- iSCREAM, SCREAM, Midori-64 [Todo, Leander, Sasaki 2016]
- Simpira v1 [Rønjom 2016]
- Haraka v.0 [Jean 2016]
- NORX v2.0 [Chaigneau et al. 2017]

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$LS(g)$ is a subspace of **linear structures** and $W_L(c)$ is the **minimal** L**-invariant subspace** containing c.

Need that $W_L(D) \subseteq LS(g)$ where D is a set of differences of keys.

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The following lightweight ciphers are resistant against invariant attacks.

- *Skinny*-64,
- *Prince*,
- $Mantis_7$

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Goal of the paper

Provide useful generalizations of nonlinear invariant attacks.

Summary of the talk

Generalized Nonlinear Invariants

Main idea

Look for a nonlinear Boolean function g and a pair $a_1, a_2 \in GF(2)^n$, such that $g(x \oplus a_1) \oplus g(F_{k_i}(x) \oplus a_2) = \text{const.} \quad \forall x.$

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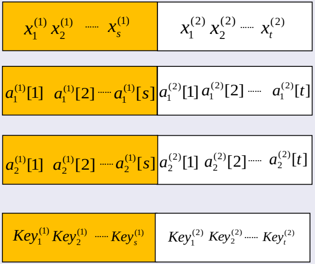
Look for a nonlinear Boolean function g and a pair $a_1, a_2 \in GF(2)^n$, such that $g(x \oplus a_1) \oplus g(F_{k_i}(x) \oplus a_2) = \text{const.} \quad \forall x$.

These are called, **generalized nonlinear invariants**, where $F_{k_i}(x)$ is a round function. For any function F, let us denote by

$$U(F, a_1, a_2) := \{g : F_2^m \to F_2 | g(x \oplus a_1) = g(F(x) \oplus a_2) \oplus c\}$$

We assume that the nonlinear terms of $g(x)$ only cover the first s input variables, and the remaining t variables have a linear relation, i.e $g(x) = f(x^{(1)}) \oplus l(x^{(2)})$.

If the round subkeys Key_j and the constants $a_i, (i = 1, 2)$ satisfy any one of the following two conditions:



(1) $a_1^{(1)} = \mathbf{0}, a_2^{(1)} \oplus Key_j^{(1)} = \mathbf{0}$;

(2) $a_1^{(1)} \neq \mathbf{0}, a_1^{(1)} \oplus a_2^{(1)} \oplus Key_j^{(1)} = \mathbf{0}$,

The generalized nonlinear invariant attack can work on the full-round block cipher.

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Moreover, we have

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Moreover, we have

$$g(P \oplus a_1) \oplus g(C \oplus a_1) = Constant'.$$

Generalized Nonlinear Invariant Attack

Distinguishing Attack by using *Generalized Nonlinear Invariant Attack*

Assume that $(P_i, C_i), (i = 1, ..., N)$ *are* N *pairs of plaintexts and ciphtexts. In a known-plaintext attack scenario, the adversary can easily determine whether* $g(P) \oplus g(C)$ *(or* $g(P \oplus a_1) \oplus g(C \oplus a_1)$ *) is constant or not for all pairs. It is clear that any random permutation has this property with a probability of* 2^{1-N} *if* g *is balanced.*

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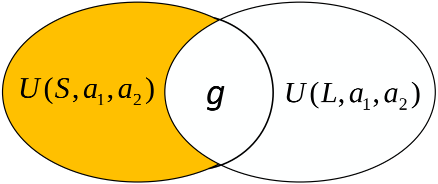
- If $\mathcal{L}$ can be viewed as an orthogonal matrix and $\deg(g) = 2$ then one can easily specify invariant for a whole round.



Assumptions-more formally

Theorem

Assume that $g_i \in U(S, a_1, a_2)$, $i = 1, \ldots, m$ are arbitrary generalized nonlinear invariants of a given S-box. Define

$$g_S(x_1, \ldots, x_m) = \sum_{i=1}^{m} \beta_i g_i(x_i), \quad \beta_i \in GF(2),$$

which is a **generalized nonlinear invariant of entire S-box layer**.

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Theorem

For SPN network if L is an **orthogonal matrix** *$M \in GF(2)^{m \times m}$ and $g' \in U(S, a_1, a_2)$ is* **quadratic**, *then $g(x_1, \ldots, x_m) = \sum_{i=1}^{m} g'(x_i)$ is also a generalized nonlinear invariant for the round function $L \circ \mathcal{S}$.*

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Remark

If some nonlinear term of g involves a nonzero bit of the round constant $c*$, then the classical invariant attack becomes rather inefficient. A pair of constants (a_1, a_2), can be helpful for eliminating the impact of this !!

Summary of the talk

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A large dimension of $W_L(D)$ should prevent from invariant attacks (regardless of S layer)?! When $W_L(D) \geq n-1$ block cipher is provably resistant against these attacks.

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Definition

For any S-box define the closed-loop invariant $CLI(S)$ as the following set

$$\{(g_1, g_2) : g_1(x) \oplus g_2(S(x)) = c_1, g_2(x) \oplus g_1(S(x)) = c_2, c_i \in GF(2)\}$$

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- $CLI(S)$ is a linear subspace

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For any S-box define the closed-loop invariant $CLI(S)$ as the following set

$$\{(g_1, g_2) : g_1(x) \oplus g_2(S(x)) = c_1, g_2(x) \oplus g_1(S(x)) = c_2, c_i \in GF(2)\}$$

- $CLI(S)$ is a linear subspace
- For every $g \in U(S)$, $(g, g) \in CLI(S)$ and $(g, 1 \oplus g) \in CLI(S)$

Is the BCLR criterion optimal?

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- $CLI(S)$ is a linear subspace
- For every $g \in U(S)$, $(g, g) \in CLI(S)$ and $(g, 1 \oplus g) \in CLI(S)$
- Usually there are more elements in $CLI(S)$ than those induced by standard invariants !!

Midori64

Midori64 uses an SPN structure and a very simple key schedule. The initial state of Midori64 can be seen as a 4×4-nibble array. A 64-bit plaintext is first input into the initial state and the key pre-whitening operation is performed. Then the state is iteratively operated 16 times with the round function. At last, the state is XORed with the post whitening key.

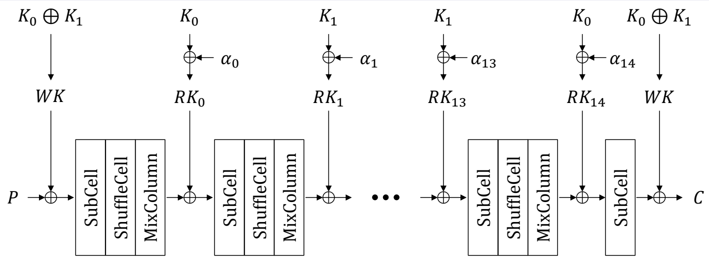

Figure: The structure of Midori64

The variant of Midori64

Construction

The Midori64 variant shares the same round function and key schedule scheme as the original Midori64. However, the only different place is that the round constants are selected from the following parameters:

Let $\alpha_i^* = ({\alpha^*}_i^1 || ... || {\alpha^*}_i^{16}), {\alpha^*}_i^j \in GF(2)^4, (i = 0, 1, ..., 14), (j = 1, ..., 16).$

(1)If $i \bmod 2 = 1$, *the 1st and 3rd bits of* ${\alpha^*}_i^j$ *are always 0.*

(2)If $i \bmod 2 = 0$, α_i^* *can choose random round constants.*

The Closed-loop Invariant for Midori64

1. For the S-box of Midori64, we can find the Closed-loop invariant below.

$$\begin{cases} g_1'(x[4],\ldots,x[1]) \oplus g_2'(y[4],\ldots,y[1]) = 1 \\ g_2'(x[4],\ldots,x[1]) \oplus g_1'(y[4],\ldots,y[1]) = 1 \\ g_1' = x[1] \oplus x[2] \oplus x[4] \oplus x[1]x[3] \\ g_2' = y[1] \oplus y[3] \end{cases}$$

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2. The linear layer of Midori64 is selected as an orthogonal matrix operation. Therefore,

$$g_1(X) = \sum_{j=1}^{16} g_1'(x_j), g_2(X) = \sum_{t=1}^{16} g_2'(x_t)$$

are the closed-loop invariants of the round function.

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- One can specify $CLI(S)$ for the whole round of this variant of Midori-64
- Efficient distinguishing attack !!

Remark

The attack works despite the fact that in this version of Midori64 $\dim W_L(D) = 64 = n$ - **standard invariant attack does not apply !!**

Building $W_L(D)$ of large dimension

For the Midori64 variant, the round keys repeat each second round. 64-bit round constants α_i^*, for $i = 0, \ldots, 14$ may be defined so that

$$D := \{\alpha_0^* \oplus \alpha_2^*, \alpha_0^* \oplus \alpha_4^*, \ldots, \alpha_0^* \oplus \alpha_{14}^*, \alpha_1^* \oplus \alpha_3^*, \alpha_1^* \oplus \alpha_5^*, \ldots, \alpha_1^* \oplus \alpha_{13}^*\},$$

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Conclusion

No obvious weaknesses for the choice of round constants, $\dim W_L(D) = n$ protects against standard invariant attacks, BUT attack based on *CLI* still applies !!

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Using computer simulations one can verify that PRESENT, PRINCE and Lblock are **resistant** against (CLI) generalized invariant attacks.

Summary of the talk

1. Overview
2. Generalized Nonlinear Invariants
3. Closed Loop Invariants
4. Conclusions

There are more generalizations and attempts to unify the work on invariant attacks. For instance, Beyne [2018] proposed a unified study within the framework of correlation matrices giving more insight towards a general structure.

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Current work: Further generalization of the concept and deeper theoretical analysis !!

Merci beaucoup!