

Generating Graphs Packed With Paths

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Overview

Motivation

Linear Cryptanalysis & Graphs

Subgraph Heuristics (for SPN)

Plots & Results

Future Work

Motivation

[BS90]

$$\mathbb{P}_x[E_k(x) + \nabla = E_k(x + \Delta)]$$

[Mat93]

$$\mathbb{P}_x[\langle \alpha, x \rangle = \langle \beta, E_k(x) \rangle]$$

[BS90]

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[Mat93]

$$\mathbb{P}_x[\langle \alpha, x \rangle = \langle \beta, E_k(x) \rangle]$$

Differential and Linear Distinguishers

In this presentation, focus on linear cryptanalysis
(differential largely analogous)

[MY92], [Mat93]

$$\mathbb{P}_x[\langle \alpha, x \rangle = \langle \beta, E_k(x) \rangle]$$

$$E_k = E_{k_r}^{(r)} \circ \dots \circ E_{k_2}^{(2)} \circ E_{k_1}^{(1)}$$

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$$U = (\alpha = u_0, \dots, u_r = \beta)$$

Iterated Ciphers and Trails

$$E_k = E_{k_r}^{(r)} \circ \dots \circ E_{k_2}^{(2)} \circ E_{k_1}^{(1)}$$

$$U = (\alpha = u_0, \dots, u_r = \beta)$$

$$C_{(u_i, u_{i+1})}^{k_i}(i) = 2 \cdot \mathbb{P}_{x \in \mathbb{F}^n}[\langle u_i, x \rangle = \langle u_{i+1}, E_{k_i}^{(i)}(x) \rangle] - 1$$

Correlation contribution for linear trail¹:

$$C_U^k = \prod_{i=0}^r C_{(u_i, u_{i+1})}^{k_i}(i)$$

¹under 'Markov cipher assumption'

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$$C_{\alpha, \beta}^k = \sum_{U: (u_0, u_r) = (\alpha, \beta)} C_U^k$$

¹under 'Markov cipher assumption'

Hull; Expected Linear Potential

For key-alternating ciphers (key-addition in the field):

$$\forall k : (C_U^k)^2 = (C_U)^2 = \prod_{i=0}^r (C_{(u_i, u_{i+1})}^k(i))^2$$

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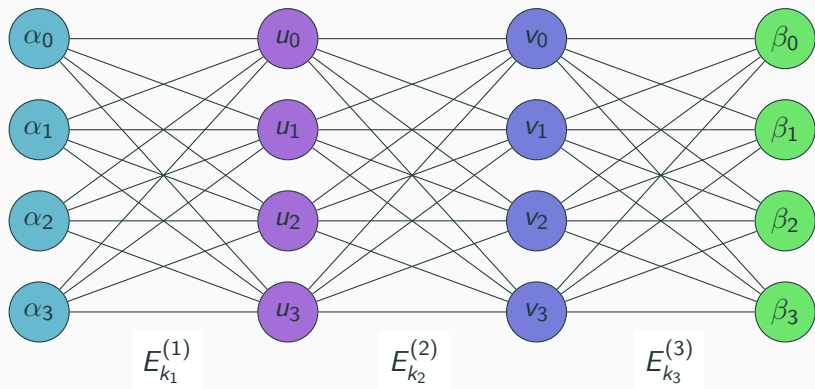
$$\forall k : (C_U^k)^2 = (C_U)^2 = \prod_{i=0}^r (C_{(u_i, u_{i+1})}^k(i))^2$$

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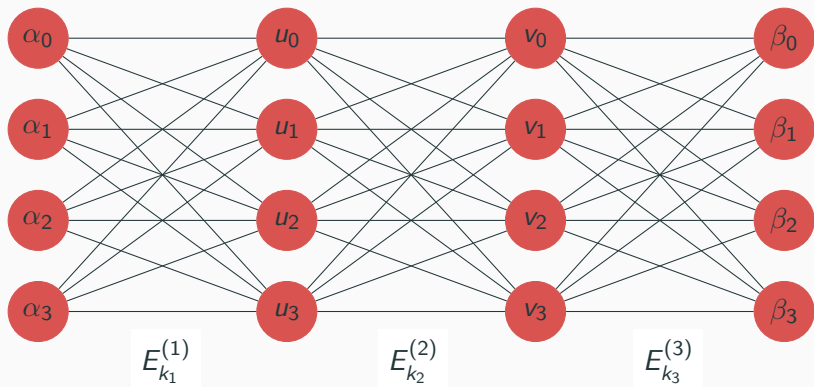
Problem: Current methods usually linear in the number of trails

Linear Cryptanalysis & Graphs

Multistage Graph



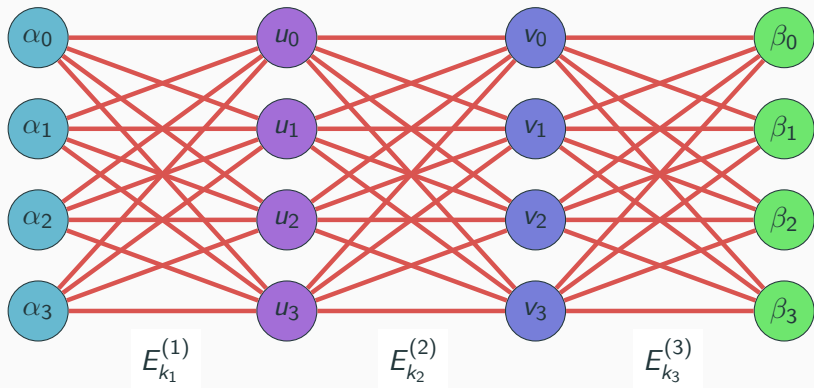
Nodes and Parities



Nodes $\alpha \in \mathbb{F}^n$ represent parities α^* for linear cryptanalysis:

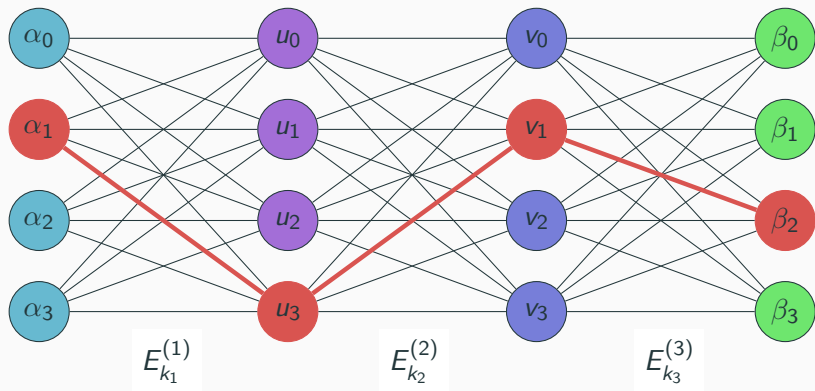
$$\alpha^* : v \mapsto \langle v, \alpha \rangle$$

Edges and Approximations



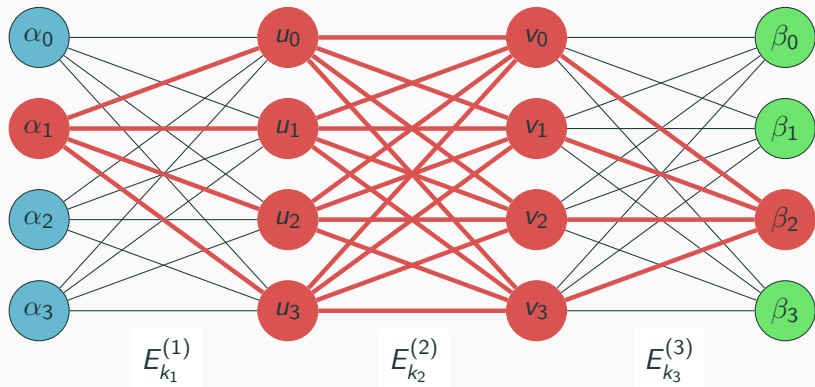
$$I(u \rightarrow v) = (C_{(u,v)}^k)^2$$

Paths and Trails



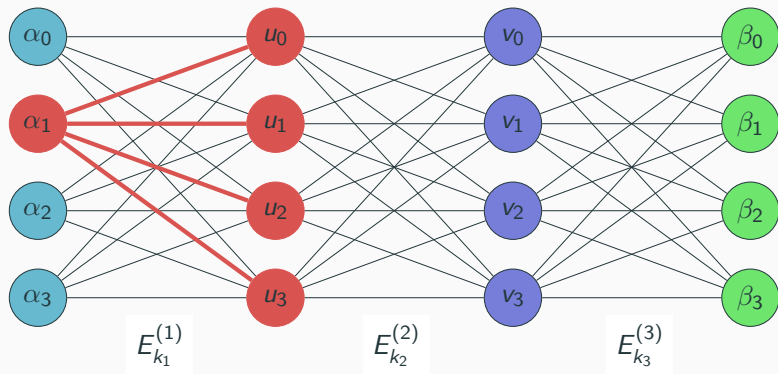
$$l(v_0 \rightsquigarrow v_r) = \prod_{i=0}^{r-1} l(v_i \rightarrow v_{i+1})$$

Hulls as Sets of Paths



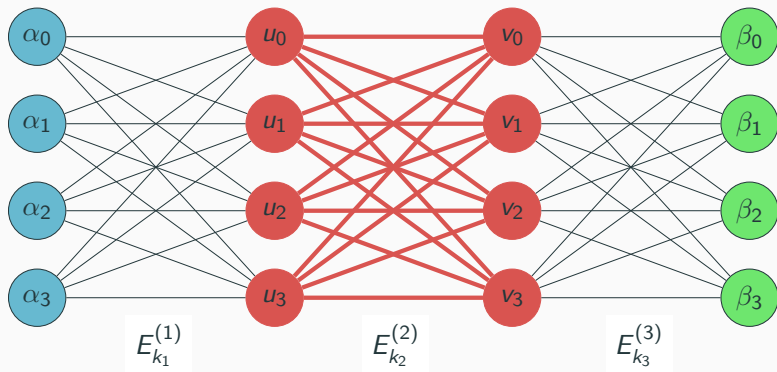
$$w_{G_{\mathcal{E}}}(\alpha \diamond \beta) = \sum l(\alpha \rightsquigarrow \beta) = \sum_v w_{G_{\mathcal{E}}}(\alpha \diamond v) \cdot l(v \rightarrow \beta)$$

Hulls as Sets of Paths



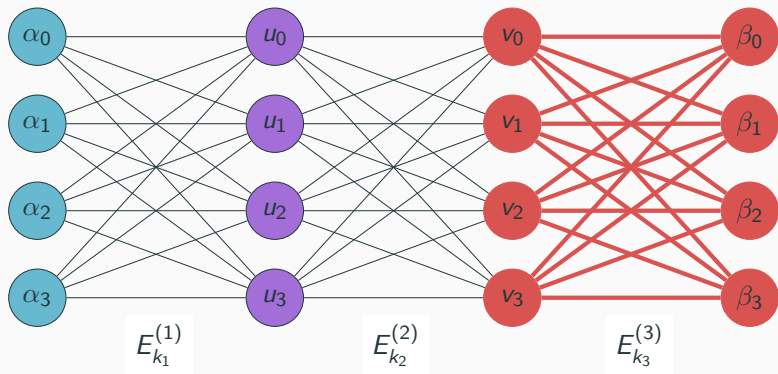
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The full graph $G_{\mathcal{E}}$ is too large.
(exponential in the block-size)

Can we find suitable $\bar{G}_\varepsilon \subset G_\varepsilon$, that contains the good trails?

i.e. $\max_{\alpha, \beta} w_{\bar{G}_\varepsilon}(\alpha \diamond \beta)$ is large.

Subgraph Heuristics (for SPN)

Overall Method

1. Pick disjoint 'families' of edges

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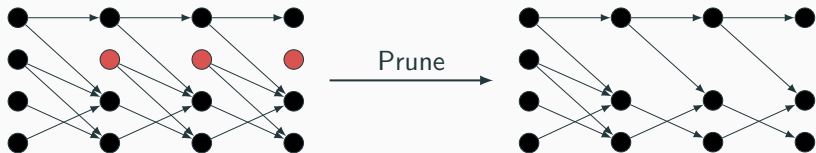
1. Pick disjoint 'families' of edges
2. Prune the families an 'approximate' graph
3. Expand the families to a full graph
4. Remove unneeded vertices & edges in resulting graph

Pruning



$$I(v \rightarrow u) = 0$$

Pruning



Example: 16-bit SPN, with four identical 4-bit S-Boxes.

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$$\rho = (1, 2^{-2}, 1, 2^{-2})$$

S-Box Patterns / Families of edges

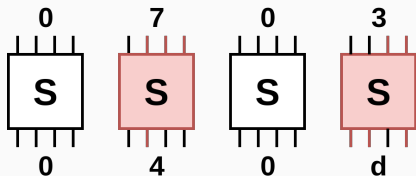
Example: 16-bit SPN, with four identical 4-bit S-Boxes.

$$C^2(0x3, 0xd) = 2^{-2}$$

$$C^2(0x7, 0x4) = 2^{-2}$$

$$p = (1, 2^{-2}, 1, 2^{-2})$$

$$\text{Ex}(p) = \{(0x0303, 0x0d0d), (0x0307, 0x0d04), \\ (0x0703, 0x040d), (0x0707, 0x0404)\}$$

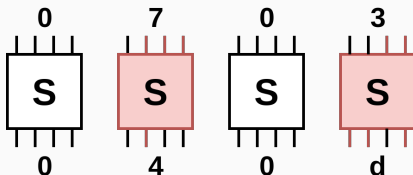


S-Box Patterns / Families of edges

$$Ex(p) = \{(0x0303, 0x0d0d), (0x0307, 0x0d04), \\ (0x0703, 0x040d), (0x0707, 0x0404)\}$$

$$Ex_{in}(p) = \{0x0303, 0x0307, 0x0703, 0x0707\}$$

$$Ex_{out}(p) = \{0x0d0d, 0x0d04, 0x040d, 0x0404\}$$



Graph Defined By S-Box Pattern Set

Given a set of S-Box patterns \mathcal{P} , the graph defined by \mathcal{P} :

$$E = \text{Ex}(\mathcal{P}) = \bigcup_{p \in \mathcal{P}} \text{Ex}(p)$$

$$V = \text{Ex}_{in}(\mathcal{P}) \cup \text{Ex}_{out}(\mathcal{P})$$

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Let \mathcal{P} be a set of S-Box patterns defining our subgraph.

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For intermediate stages:

$$v \notin \text{Ex}_{in}(\mathcal{P}) \cap \text{Ex}_{out}(\mathcal{P}) \implies v \text{ is pruned}$$

Problem: $\text{Ex}(\mathcal{P})$ too large to store explicitly ($|\text{Ex}(\mathcal{P})| \gg |\mathcal{P}|$)

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Idea: Can we prune \mathcal{P} before expanding?

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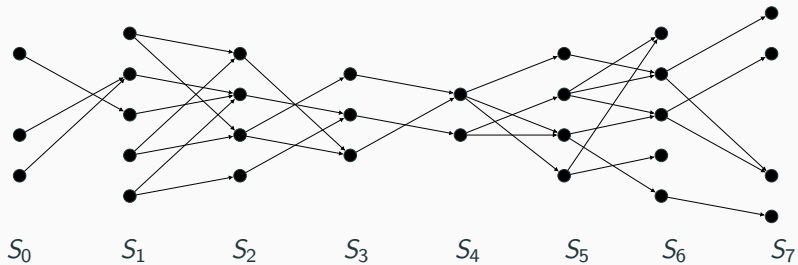
Generate an approximation of $\bar{G}_{\mathcal{E}} = \text{Ex}(\mathcal{P})$, by applying a compression function $g_j : \mathbb{F}^n \rightarrow \mathbb{F}^{n/j}$ to every vertex.

$$u \rightarrow v \in \bar{G}_{\mathcal{E}} \implies \hat{g}_j(u) \rightarrow \hat{g}_j(v) \in \hat{g}_j(\bar{G}_{\mathcal{E}})$$

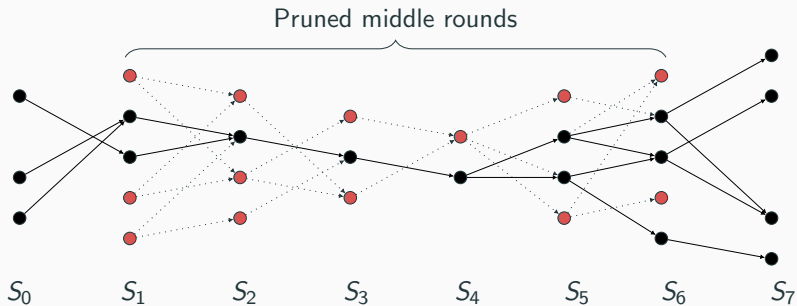
Iteratively refine the compression:

1. Generate a set of patterns \mathcal{P} .
2. Pick a $j > 1$ such that j is a power of two:
 - 2.1 Generate the graph $\hat{g}_j(\bar{G}_{\mathcal{E}})$ from \mathcal{P} and prune.
 - 2.2 Remove dead patterns from \mathcal{P} according to $\hat{g}_j(\bar{G}_{\mathcal{E}})$.
 - 2.3 If $j = 2$ then stop. Otherwise set $j = j/2$ and repeat.

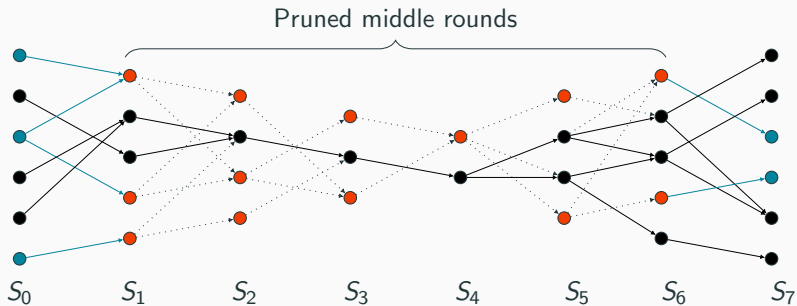
Vertex Anchoring



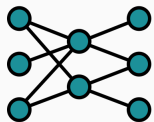
Vertex Anchoring



Vertex Anchoring



Plots & Results

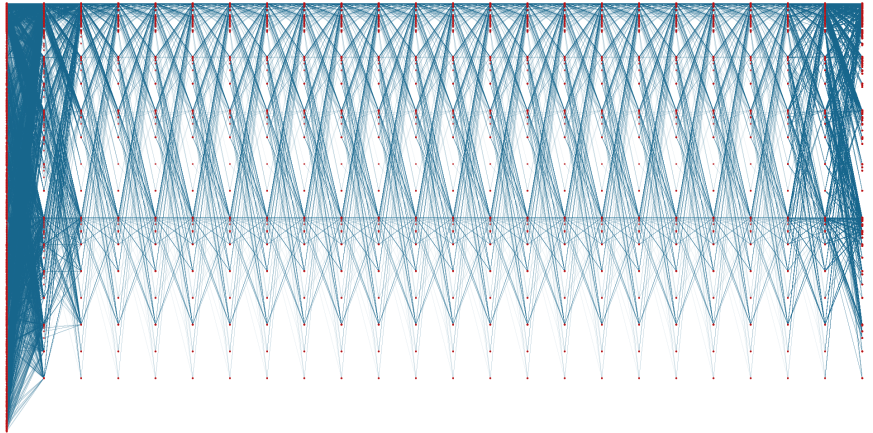


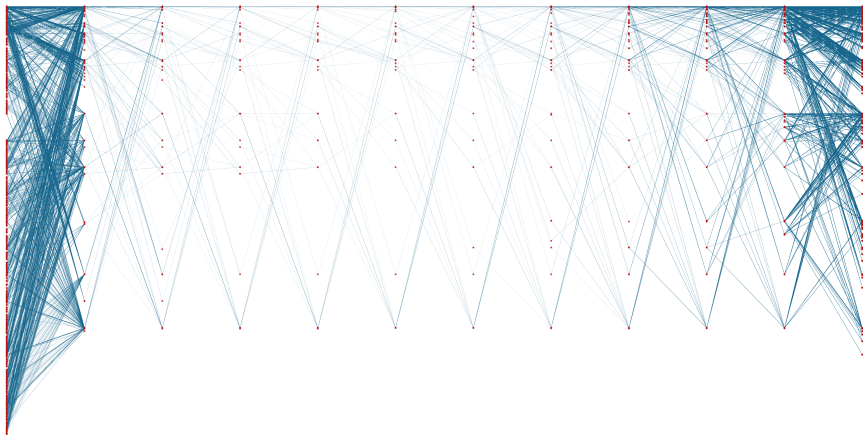
cryptagraph

<https://gitlab.com/psve/cryptagraph>

Plots of subgraphs (for small parameters)

PRESENT [BKL⁺07]





Linear Results

Cipher (Total rounds, block size)	Rounds	$ \mathcal{A} $	a	$ \alpha \succ \beta $	ELP	T_E	T_S
AES [eST01] (10, 128)	3	$2^{29.9}$	$2^{24.0}$	2^1	$2^{-53.36}$	0.0	0.0
	4	$2^{38.8}$	$2^{24.0}$	2^4	$2^{-147.88}$	2.5	20.0
EPCBC-48 [YKPH11] (32, 48)	15 † [Bul13]	$2^{26.1}$	–	$2^{31.3}$	$2^{-43.74}$	0.0	0.4
	16 † [Bul13]	$2^{26.1}$	–	$2^{34.0}$	$2^{-46.77}$	0.0	0.4
EPCBC-96 [YKPH11] (32, 96)	31	$2^{27.6}$	–	$2^{63.6}$	$2^{-94.47}$	0.0	0.4
	32	$2^{27.6}$	–	$2^{63.6}$	$2^{-97.59}$	0.0	0.4
FLY [KG16] (20, 64)	8	$2^{32.5}$	–	$2^{6.5}$	$2^{-54.83}$	0.1	6.0
	9	$2^{32.5}$	–	$2^{6.1}$	$2^{-63.00}$	0.2	8.8
GIFT-64 [BPP+17] (28, 64)	11	$2^{31.8}$	–	$2^{5.1}$	$2^{-55.00}$	0.1	8.0
	12	$2^{32.7}$	–	$2^{3.6}$	$2^{-64.00}$	0.2	41.5
KHAZAD [BR00] (8, 64)	2	$2^{18.3}$	$2^{25.0}$	2^0	$2^{-37.97}$	0.0	0.0
	3	$2^{30.1}$	$2^{25.0}$	2^0	$2^{-68.01}$	0.2	0.2
KLEIN [GNL11] (12, 64)	5	$2^{30.8}$	$2^{17.0}$	2^0	$2^{-46.0}$	0.0	0.0
	6	$2^{39.6}$	$2^{16.9}$	2^0	$2^{-66.0}$	0.3	0.0
LED [GPPr11] (32, 64)	4	$2^{24.7}$	2^{25}	2^2	$2^{-48.68}$	0.0	0.9
MANTIS, [BJK+16] (2 · 8, 64)	2 · 4	$2^{34.3}$	$2^{24.0}$	$2^{15.0}$	$2^{-49.05}$	0.1	0.0
Midori64 [BBI+15] (16, 64)	6	$2^{44.3}$	–	$2^{19.0}$	$2^{-53.02}$	25.9	0.8
	7	$2^{46.5}$	–	$2^{21.9}$	$2^{-62.88}$	53.1	5.5
PRESENT [BKL+07] (31, 64)	23 † [Ohk09]	$2^{31.1}$	–	$2^{55.0}$	$2^{-61.00}$	0.1	6.8
	24 † [Ohk09]	$2^{31.1}$	–	$2^{57.9}$	$2^{-63.61}$	0.1	6.9
	25 † [Ohk09]	$2^{31.1}$	–	$2^{60.7}$	$2^{-66.21}$	0.1	6.9
PRIDE [ADK+14] (20, 64)	15	$2^{27.1}$	–	2^0	$2^{-58.00}$	0.0	0.0
	16	$2^{37.4}$	–	2^3	$2^{-63.99}$	1.8	0.0
PRINCE [BCG+12] (2 · 6, 64)	2 · 3	$2^{18.1}$	–	$2^{2.0}$	$2^{-54.00}$	0.0	0.0
	2 · 4	$2^{38.3}$	–	$2^{6.8}$	$2^{-63.82}$	2.1	0.4
PUFFIN [CHW08] (32, 64)	32	$2^{26.8}$	–	$2^{112.4}$	$2^{-51.90}$	0.0	0.0
QARMA [Ava17] (2 · 8, 64)	2 · 3	$2^{24.8}$	$2^{24.0}$	$2^{5.0}$	$2^{-53.71}$	0.0	0.0
RECTANGLE [ZBL+14] (25, 64)	12 † [ZBL+14]	$2^{31.1}$	–	$2^{15.0}$	$2^{-52.27}$	0.1	21.1
	13 † [ZBL+14]	$2^{31.1}$	–	$2^{15.9}$	$2^{-58.14}$	0.1	25.9
	14 † [ZBL+14]	$2^{31.1}$	–	$2^{18.3}$	$2^{-62.96}$	0.1	31.1
SKINNY-64 [BJK+16] (32, 64)	8	$2^{41.4}$	$2^{23.7}$	$2^{34.4}$	$2^{-50.46}$	0.7	50.7
	9	$2^{41.4}$	$2^{23.9}$	$2^{31.3}$	$2^{-69.83}$	0.4	8.9

Differential Results

Cipher (Total rounds, block size)	Rounds	$ D $	a	$ \Delta \diamond \nabla $	EDP	T_g	T_s
AES [sT01] (10, 128)	3	$2^{18.7}$	$2^{24.0}$	2^0	$2^{-54.00}$	0.0	0.0
	4	$2^{36.9}$	$2^{24.0}$	2^0	$2^{-150.00}$	0.7	0.3
EPBCB-48 [YKPH11] (32, 48)	13	$2^{28.4}$	–	$2^{21.2}$	$2^{-43.86}$	0.1	13.7
	14	$2^{28.4}$	–	$2^{20.4}$	$2^{-47.65}$	0.1	14.0
EPBCB-96 [YKPH11] (32, 96)	20	$2^{32.8}$	–	$2^{16.9}$	$2^{-92.73}$	1.1	21.6
	21	$2^{32.8}$	–	$2^{19.9}$	$2^{-97.78}$	1.1	22.6
FLY [KG16] (20, 64)	8	$2^{31.6}$	–	$2^{4.9}$	$2^{-55.76}$	0.1	2.6
	9	$2^{33.2}$	–	$2^{7.3}$	$2^{-63.35}$	0.2	17.8
GIFT-64 [BPP ⁺ 17] (28, 64)	12 † [ZDY18]	$2^{22.4}$	–	$2^{3.3}$	$2^{-56.57}$	0.0	0.0
	13	$2^{22.4}$	–	$2^{3.6}$	$2^{-60.42}$	0.0	0.0
KHAZAD [BR00] (8, 64)	2	$2^{25.8}$	$2^{24.8}$	2^0	$2^{-45.42}$	0.0	0.0
	3	$2^{25.8}$	$2^{25.0}$	2^0	$2^{-81.66}$	0.0	0.0
KLEIN [GNL11] (12, 64)	5	$2^{30.8}$	$2^{17.0}$	$2^{1.0}$	$2^{-45.91}$	0.0	0.0
	6	$2^{39.7}$	$2^{24.0}$	$2^{1.0}$	$2^{-69.00}$	0.3	6.4
LED [GPPR11] (32, 64)	4	$2^{37.7}$	$2^{24.0}$	2^1	$2^{-49.42}$	0.5	0.1
MANTIS ₇ [BJK ⁺ 16] (2 · 8, 64)	2 · 4	$2^{37.7}$	–	$2^{18.6}$	$2^{-47.98}$	0.9	0.1
Midori64 [BBI ⁺ 15] (16, 64)	6	$2^{42.2}$	$2^{23.9}$	$2^{19.6}$	$2^{-52.37}$	1.6	1.0
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PRESENT [BKL ⁺ 07] (31, 64)	15	$2^{30.3}$	–	$2^{27.2}$	$2^{-58.00}$	0.1	16.2
	16 † [Abd12]	$2^{30.3}$	–	$2^{28.9}$	$2^{-61.80}$	0.1	18.0
PRIDE [ADK ⁺ 14] (20, 64)	17	$2^{30.3}$	–	$2^{32.9}$	$2^{-63.52}$	0.1	18.8
	15	$2^{35.9}$	$2^{23.6}$	$2^{5.0}$	$2^{-58.00}$	0.5	36.5
PRINCE [BCG ⁺ 12] (2 · 6, 64)	16	$2^{35.9}$	$2^{23.6}$	$2^{17.4}$	$2^{-63.99}$	0.5	44.1
	2 · 3 † [CFG ⁺ 14]	$2^{14.0}$	2^{19}	2^1	$2^{-55.91}$	0.0	0.0
PUFFIN [CHW08] (32, 64)	2 · 4	$2^{38.7}$	–	$2^{9.0}$	$2^{-67.32}$	3.0	1.0
	32	$2^{26.0}$	–	$2^{63.7}$	$2^{-59.63}$	0.0	0.0
QARMA [Ava17] (2 · 8, 64)	2 · 3	$2^{24.8}$	$2^{26.0}$	$2^{7.3}$	$2^{-56.47}$	0.1	0.0
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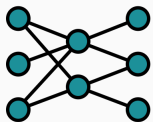
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	25 † [Ohk09]	$2^{31.1}$	–	$2^{60.7}$	$2^{-66.21}$	0.1	6.9
PUFFIN [CHW08] (32, 64)	32	$2^{26.8}$	–	$2^{112.4}$	$2^{-51.90}$	0.0	0.0
RECTANGLE [ZBL ⁺ 14] (25, 64)	12 † [ZBL ⁺ 14]	$2^{31.1}$	–	$2^{15.0}$	$2^{-52.27}$	0.1	21.1
	13 † [ZBL ⁺ 14]	$2^{31.1}$	–	$2^{15.9}$	$2^{-58.14}$	0.1	25.9
	14 † [ZBL ⁺ 14]	$2^{31.1}$	–	$2^{18.3}$	$2^{-62.98}$	0.1	31.1
Cipher (Total rounds, block size)	Rounds	$ \mathcal{D} $	a	$ \Delta \diamond \nabla $	EDP	T_g	T_s
EPCBC-48 [YKPH11] (32, 48)	13	$2^{28.4}$	–	$2^{21.2}$	$2^{-43.86}$	0.1	13.7
	14	$2^{28.4}$	–	$2^{20.4}$	$2^{-47.65}$	0.1	14.0
EPCBC-96 [YKPH11] (32, 96)	20	$2^{32.8}$	–	$2^{16.9}$	$2^{-92.73}$	1.1	21.6
	21	$2^{32.8}$	–	$2^{19.9}$	$2^{-97.78}$	1.1	22.6
PRESENT [BKL ⁺ 07] (31, 64)	15	$2^{30.3}$	–	$2^{27.2}$	$2^{-58.00}$	0.1	16.2
	16 † [Abd12]	$2^{30.3}$	–	$2^{28.9}$	$2^{-61.80}$	0.1	18.0
	17	$2^{30.3}$	–	$2^{32.9}$	$2^{-63.52}$	0.1	18.8
PUFFIN [CHW08] (32, 64)	32	$2^{26.0}$	–	$2^{63.7}$	$2^{-59.63}$	0.0	0.0
RECTANGLE [ZBL ⁺ 14] (25, 64)	13 † [ZBL ⁺ 14]	$2^{31.1}$	–	$2^{15.3}$	$2^{-55.64}$	0.1	32.2
	14 † [ZBL ⁺ 14]	$2^{31.1}$	–	$2^{15.9}$	$2^{-60.64}$	0.1	41.3
	15 † [ZBL ⁺ 14]	$2^{31.1}$	–	$2^{18.2}$	$2^{-65.64}$	0.1	50.2

Future Work

Support for ARX ciphers.

Support for ARX ciphers.

Better heuristics for Feistel networks.



cryptagraph

<https://gitlab.com/psve/cryptagraph>