Security of Even-Mansour Ciphers under Key-dependent Messages

Pooya Farshim, **Louiza Khati**, Damien Vergnaud

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Wednesday, March 7th, 2018

KDM Security

- KDM for "Key Dependent Message" [BRS03]
  - Encryption Scheme Security in Presence of Key Dependent Message
- Disk Encryption [BHHO08]
  - Circular-Secure Encryption from Decision Diffie-Hellman

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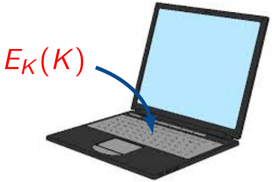
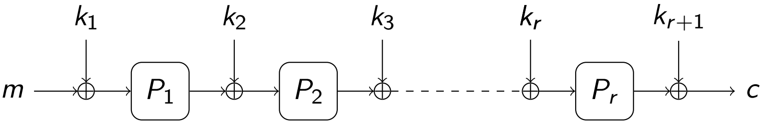


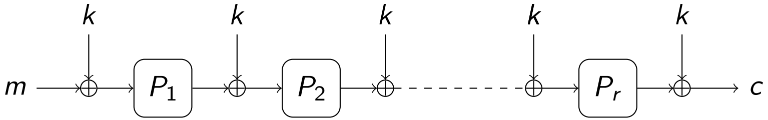
Even-Mansour r rounds



Many configurations for r rounds:

- r public random permutations: equal; **independent;** related.
- $r + 1$ keys: equal; **independent;** key schedule.

Even-Mansour r rounds



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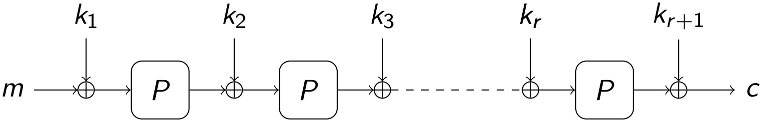

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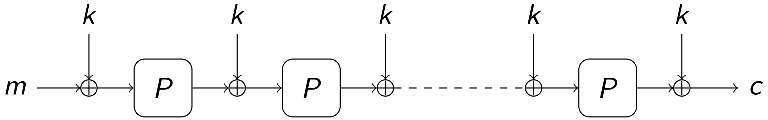

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Even-Mansour: Previous works

- Even-Mansour [EM97]
  - 1-r: SPRP up to the birthday bound
- 2-r Even Mansour [CLL$^+$14]
  - Master key, $P_1 = P_2$, 2-r EM secure beyond BB in RPM.
- Related Key Attack security [CS15], [FP15]
  - $\mathcal{A}$ can apply offset Δ to keys: $k_i \oplus \Delta$,
  - Single key, 2-r EM is xor-RKA CPA secure,
  - Single key, 3-r EM is xor-RKA CCA secure.
- Indifferentiability from an Ideal Cipher [DSST17]
  - 5-r EM necessary and sufficient.
- KDM security ?

Security Model: KDM security

$$K \xleftarrow{\$} \{0,1\}^k$$
$$b \xleftarrow{\$} \{0,1\}$$

Encryption: ϕ is a function
(including constants)

(b=1) Ideal world 

 π_E, π_D

Adversary

Oracle

(b=0) Real world 

EM_K, EM_K^-

Security Model: KDM security

Encryption: ϕ is a function
(including constants)

$$K \xleftarrow{\$} \{0,1\}^k$$
$$b \xleftarrow{\$} \{0,1\}$$

$\phi(.)$

Adversary

c

Oracle

(b=1) Ideal world 

 π_E, π_D

$$c = \pi_E(\phi(K))$$

(b=0) Real world 

EM_K, EM_K^{-}

$$c = EM_K(\phi(K))$$

Security Model: KDM security

Decryption: c is **constant**

$$K \xleftarrow{\$} \{0,1\}^k$$
$$b \xleftarrow{\$} \{0,1\}$$

c

Adversary

Oracle

m

(b=1) Ideal world 

 π_E, π_D

$$m = \pi_D(c)$$

(b=0) Real world 

EM_K, EM_K^-

$$m = EM_K^-(c)$$

Decrypting c **such that** $c = O(\phi(.))$
with $\phi(.) \neq$ **constant is forbidden!!**

KDM security requires Claw-Freeness

No restriction on set Φ

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Example: $\phi_i(.)$ = Set the i-th bit of K to 0 and Id

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Key Recovery Attack!

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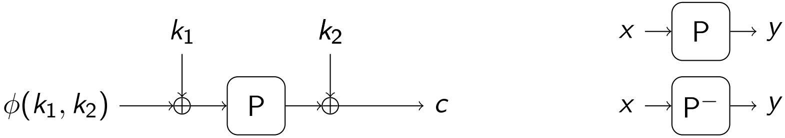
Key Recovery Attack!

Φ must be Claw-Free: It is hard to find $\phi_1 \neq \phi_2$ such that $\Pr[\phi_1(K) = \phi_2(K)]$ "is high".

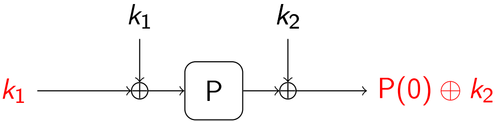
KDM security: Even-Mansour 1 round

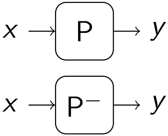
Key extraction with a claw-free set Φ



KDM Attack on 1-r Even-Mansour

Key extraction with a claw-free set Φ





k_2 extraction:

$\phi_1(k) = k_1$; $c_1 = P(0) \oplus k_2$

A computes $c_1 \oplus y_1 \rightarrow k_2$

P

$x_1 = 0; y_1 = P(0)$

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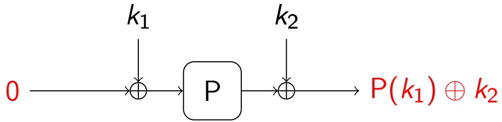
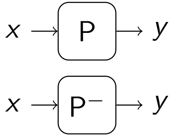




k_2 extraction:
$\phi_1(k) = k_1;\ c_1 = P(0) \oplus k_2$
A computes $c_1 \oplus y_1 \rightarrow k_2$

P
$x_1 = 0; y_1 = P(0)$

k_1 extraction:
$\phi_2(k) = 0;\ c_2 = P(k_1) \oplus k_2;$
A computes $z = k_2 \oplus c_2$
$y_2 \rightarrow k_1$

P^-
$x_2 = z; y_2 = P^-(z)$

Even-Mansour KDM security under a set Φ

r	Perm	Keys	Set Φ ind. P_i
1	P	K_1, K_2	cf, offset-free*
2	P, P	K, K, K	cf, offset-free*
2	P, P	K_1, K_2, K_3	cf, ox-free
2	P_1, P_2	K_1, K_2, K_3	cf
3	P, P, P	K, K, K, K	cf, offset-free*
3	P, P, P	K_1, K_2, K_3, K_4	cf
3	P_1, P_2, P_3	K, K, K, K	cf
n	P, P, ..., P	$K, K, \ldots, K$	cf, offset-free*

offset: (ϕ, X) such that $\phi(K_1, K_2) = K_1 \oplus X$.
ox: (ϕ, X) such that $\phi(K_1, K_2, K_3) = K_1 \oplus K_2 \oplus X$.

* Sliding attack if the set Φ is not offset-free.

Security proof

KDM security of Even-Mansour 2 rounds

- Independent random permutations: $\mathsf{P}_1^{\pm}, \mathsf{P}_2^{\pm}$
- Independent random keys: K_1, K_2, K_3

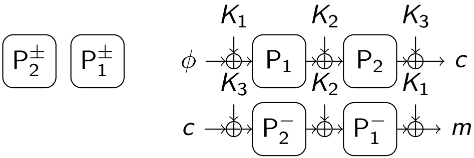
KDM rules:

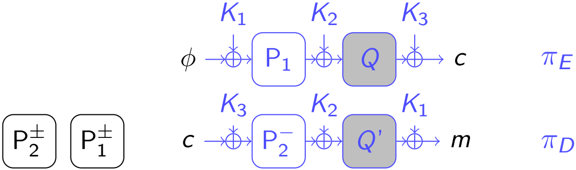
- Encryption/Decryption of oracle answers
- No repeat queries
  - $\phi_1 \neq \phi_2$
  - $c_1 \neq c_2$

The set Φ is:

- Claw-Free
- Functions ϕ independent of $\mathsf{P}_i^{\pm}$

Security proof





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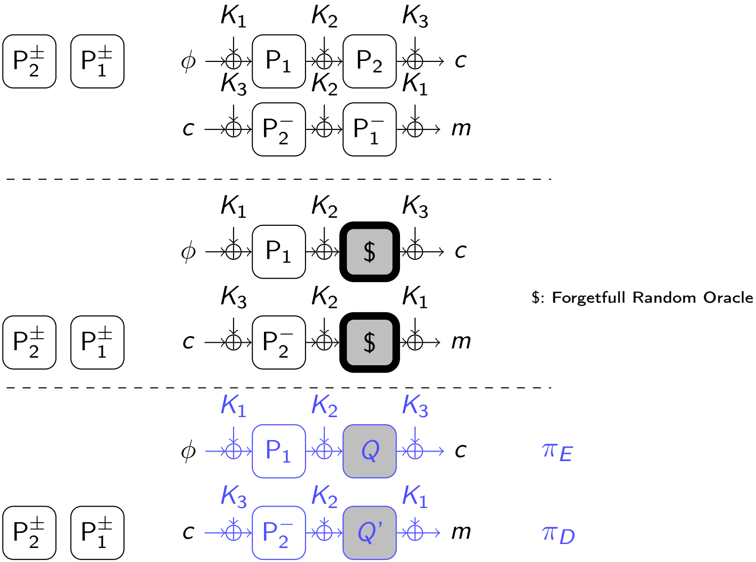

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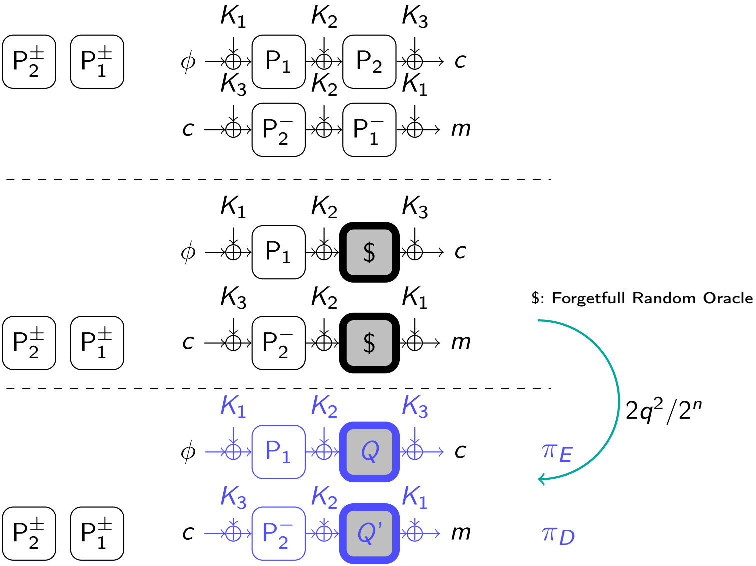

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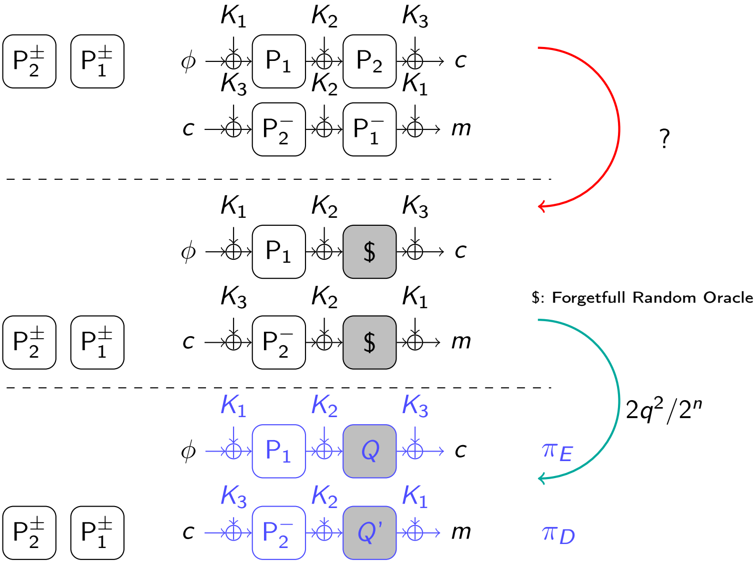
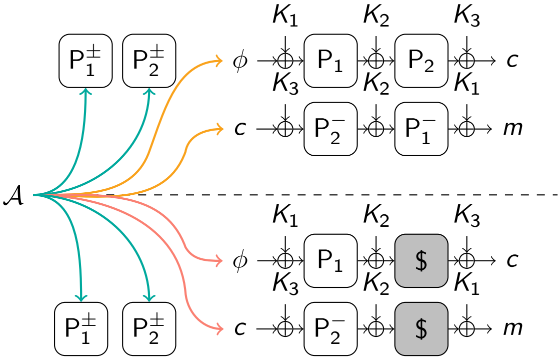


Security proof: 2-r $EM^{P_1,P_2}[K_1, K_2, K_3]$



Inconsistencies ?

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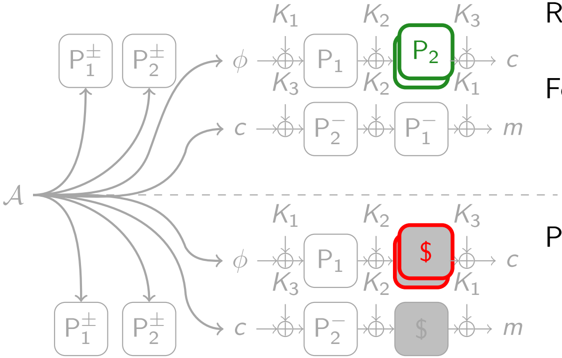

Reductions

Forgetfull game:

- no repeated queries on \$
- no "circular queries"

PRF/PRP switching lemma

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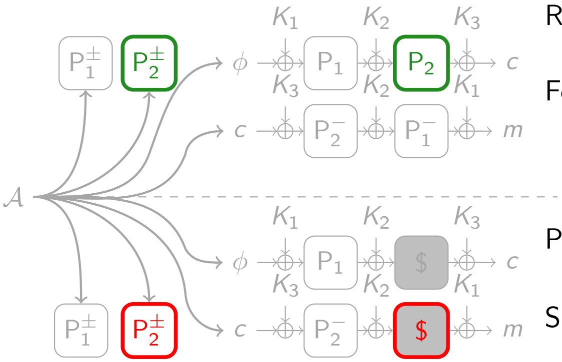

Reductions

Forgetfull game:

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- no "circular queries"

PRF/PRP switching lemma

Splitting game:

- no repeated queries on \$
- no "circular queries"

PRF/PRP switching lemma

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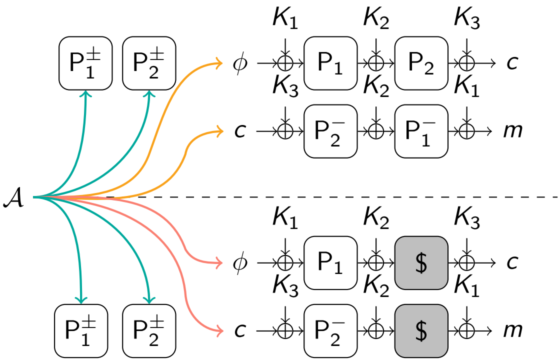

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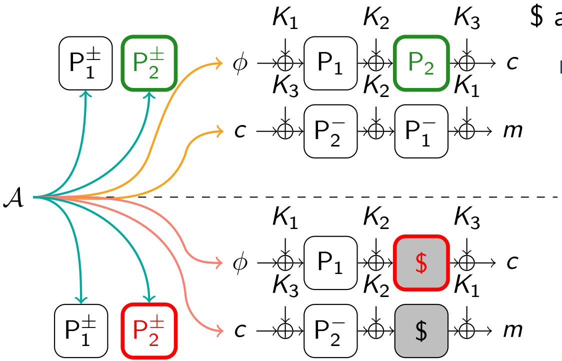

\$ and direct accessed P_i

- $Enc(\phi)$ and $P_2^{+}(X)$
  - $X = P_1(\phi(K) \oplus K_1) \oplus K_2$
  - $P_2(X) = c \oplus K_3$

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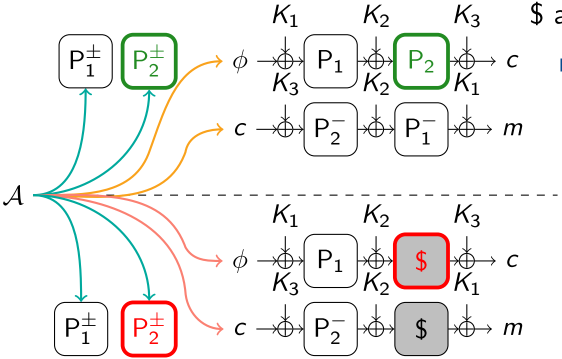

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- $Enc(\phi), P_2^-(Y)$
  - $Y = c \oplus K_3$
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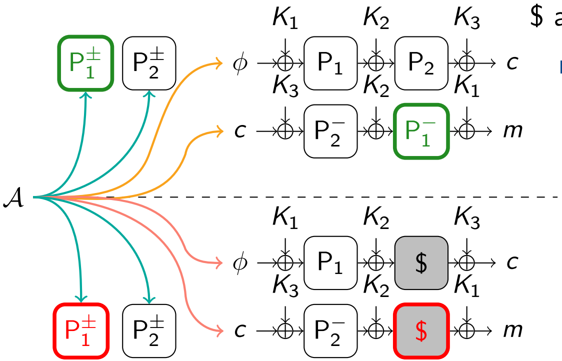

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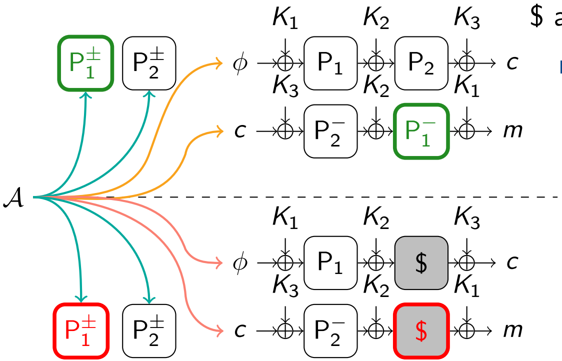
- $Enc(\phi), \mathrm{P}_2^{-}(Y)$
  - $Y = c \oplus K_3$
  - $X = \mathrm{P}_1(\phi(K) \oplus K_1) \oplus K_2$
- $Dec(c), \mathrm{P}_1^{-}(Y)$
  - $\mathrm{P}_2^{-}(c \oplus K_3) \oplus K_2 = Y$
  - $m \oplus K_1 = \mathrm{P}_1^{-}(Y)$

Security proof: 2-r $\mathrm{EM}^{\mathrm{P}_1,\mathrm{P}_2}[K_1, K_2, K_3]$



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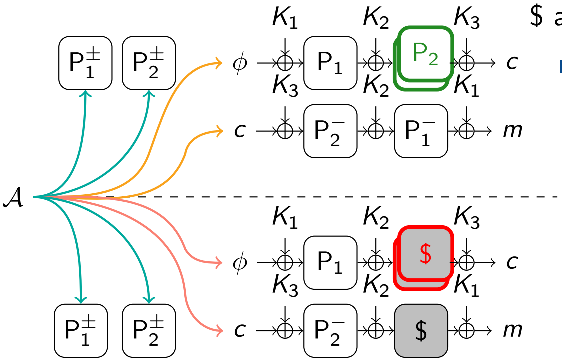
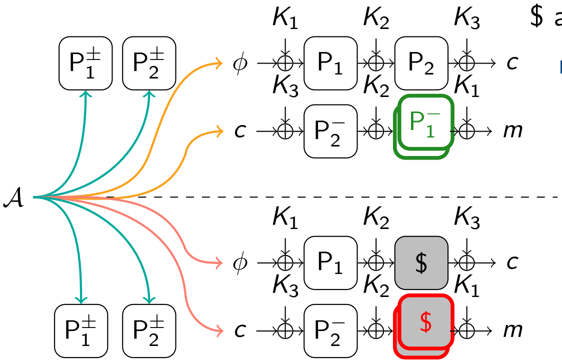


\$ and itself

- $Enc^{+}(\phi_1)$ and $Enc(\phi_2)$
  - $\phi_1(K) = \phi_2(K)$ **Claw-free**

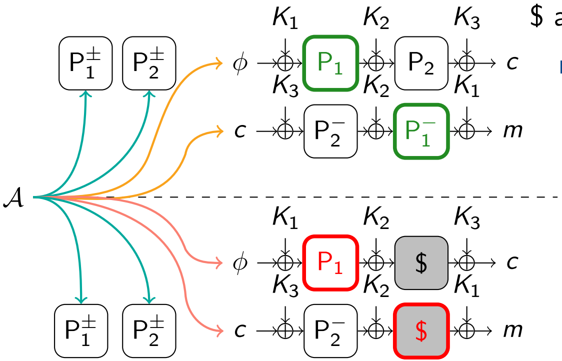
Security proof: 2-r $EM^{P_1,P_2}[K_1, K_2, K_3]$



\$ and itself

- $Enc^{+}(\phi_1)$ and $Enc(\phi_2)$
  - $\phi_1(K) = \phi_2(K)$ **Claw-free**
- $Dec(c_1), Dec(c_2)$
  - $c_1 = c_2$ repeat

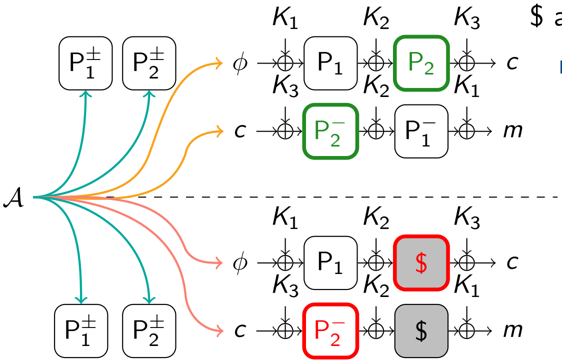
Security proof: 2-r $EM^{P_1,P_2}[K_1, K_2, K_3]$



\$ and internal P_i

- $Dec(c_1)$ then $Enc(\phi_2)$
  - $m_1 \oplus K_1 = \phi_2(K) \oplus K_1$ **Claw-free** or forbidden
  - $P_2^{-}(c_1 \oplus K_3) \oplus K_2 = P_1(\phi_2(K) \oplus K_1)$

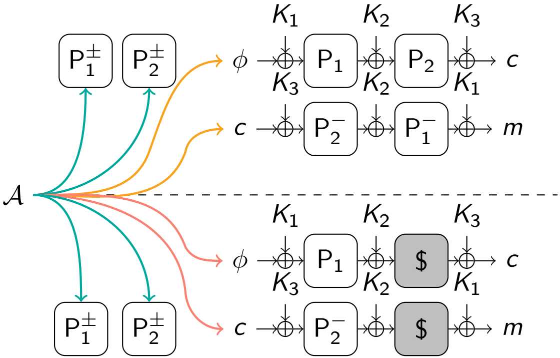
Security proof: 2-r $\mathrm{EM}^{\mathrm{P}_1,\mathrm{P}_2}[K_1, K_2, K_3]$



\$ and internal P_i

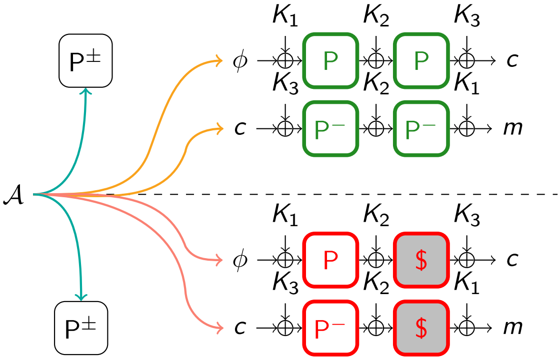
- $Dec(c_1)$ then $Enc(\phi_2)$
  - $m_1 \oplus K_1 = \phi_2(K) \oplus K_1$ **Claw-free** or forbidden
  - $\mathrm{P}_2^{-}(c_1 \oplus K_3) \oplus K_2 = \mathrm{P}_1(\phi_2(K) \oplus K_1)$
- $Enc(\phi_1)$ then $Dec(c_2)$
  - $\mathrm{P}_1(\phi_1(K) \oplus K_1) \oplus K_2 = \mathrm{P}_2^{-}(c_2 \oplus K_3)$
  - $c_1 \oplus K_3 = c_2 \oplus K_3$ forbidden

Security proof: 2-r $\mathsf{EM}^{\mathsf{P}_1,\mathsf{P}_2}[K_1, K_2, K_3]$



Φ-KDM Security up to the birthday bound with a set Φ that is Claw-Free and P_i independent.

Security proof: 2-r $EM^{P,P}[K_1, K_2, K_3]$



$P_1 = P_2 = P$

More collisions !!

Conclusion

To sum up:

- KDM security of 2 rounds Even-Mansour different keys, different permutations

In the paper:

- General framework to analyse with a two stage-adversary
  - KDM security, RKA security,
  - Different block ciphers.
- KDM security for Ideal Cipher;
- Analysis of different Even-Mansour configurations

Even-Mansour KDM security under a set Φ

r	Perm	Keys	Set Φ ind. P_i
1	P	K_1, K_2	cf, offset-free*
2	P, P	K, K, K	cf, offset-free*
2	P, P	K_1, K_2, K_3	cf, ox-free
2	$\mathsf{P}_1, \mathsf{P}_2$	K_1, K_2, K_3	cf
3	$\mathsf{P}, \mathsf{P}, \mathsf{P}$	K, K, K, K	cf, offset-free*
3	$\mathsf{P}, \mathsf{P}, \mathsf{P}$	K_1, K_2, K_3, K_4	cf
3	$\mathsf{P}_1, \mathsf{P}_2, \mathsf{P}_3$	K, K, K, K	cf
n	$\mathsf{P}, \mathsf{P}, \ldots, \mathsf{P}$	$K, K, \ldots, K$	cf, offset-free*

offset: (ϕ, X) such that $\phi(K_1, K_2) = K_1 \oplus X$.
ox: (ϕ, X) such that $\phi(K_1, K_2, K_3) = K_1 \oplus K_2 \oplus X$.

* Sliding attack if the set Φ is not offset-free.

Thank you for your attention!
Questions?

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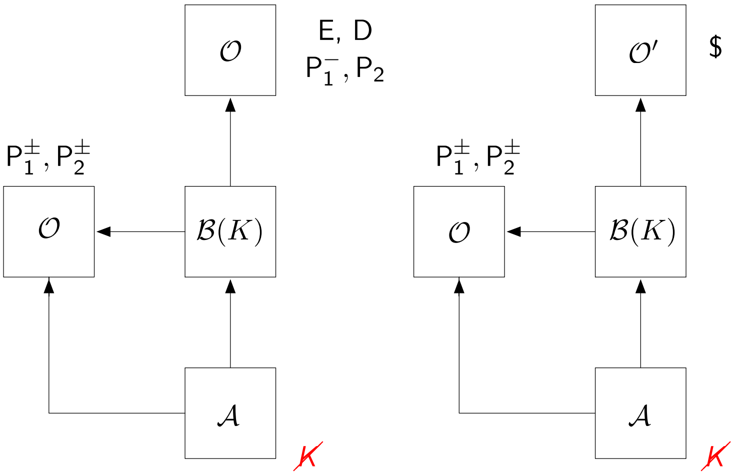
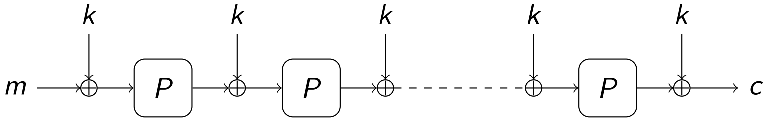
General Framework: Two-stage adversary



Sliding attack: Even-Mansour r rounds



$\phi_1 = 0$ then $EM(0) = P(k) \oplus k = c_1$
$\phi_2 = c_1 \oplus k$ then $c_2 = \mathrm{P}(\mathrm{P}(k) \oplus k) \oplus k$
$\mathcal{A}$ asks $x = c_1$ to P then $y = P(c_1) = \mathrm{P}[\mathrm{P}(k) \oplus k]$
$\mathcal{A}$ can computes k: $c_2 \oplus y$

Dan Boneh, Shai Halevi, Michael Hamburg, and Rafail Ostrovsky.
Circular-secure encryption from decision Diffie-Hellman.
In David Wagner, editor, CRYPTO 2008, volume 5157 of LNCS, pages 108–125. Springer, Heidelberg, August 2008.

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