Exponential S-Boxes: a Link Between the S-Boxes of BelT and Kuznyechik/Streebog

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Fast Software Encryption 2017

UNIVERSITÉ DU LUXEMBOURG

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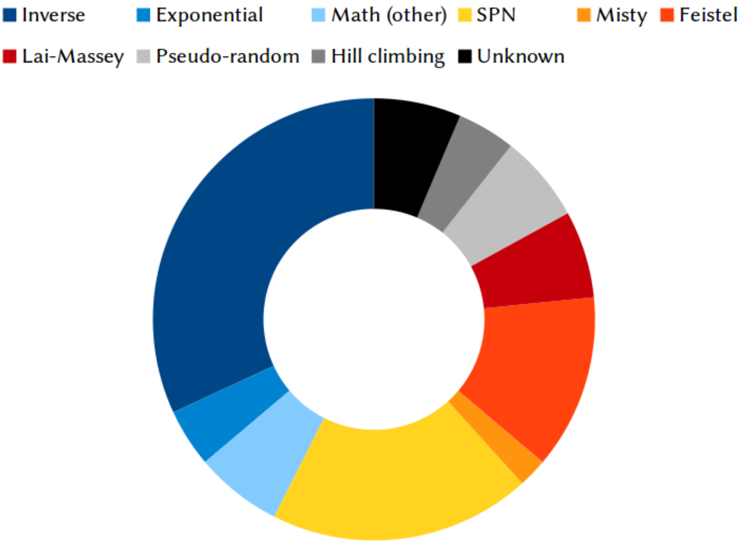
S-Box Design

■ Inverse ■ Exponential ■ Math (other) ■ SPN ■ Misty ■ Feistel
■ Lai-Massey ■ Pseudo-random ■ Hill climbing ■ Unknown

S-Box Design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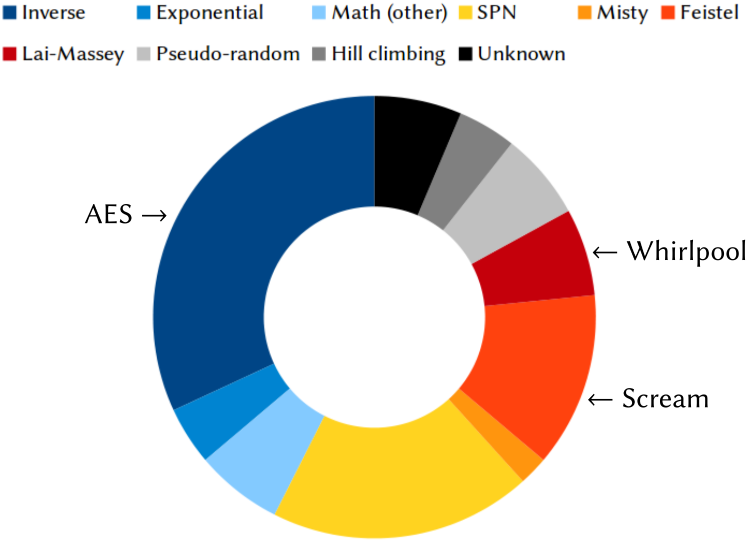
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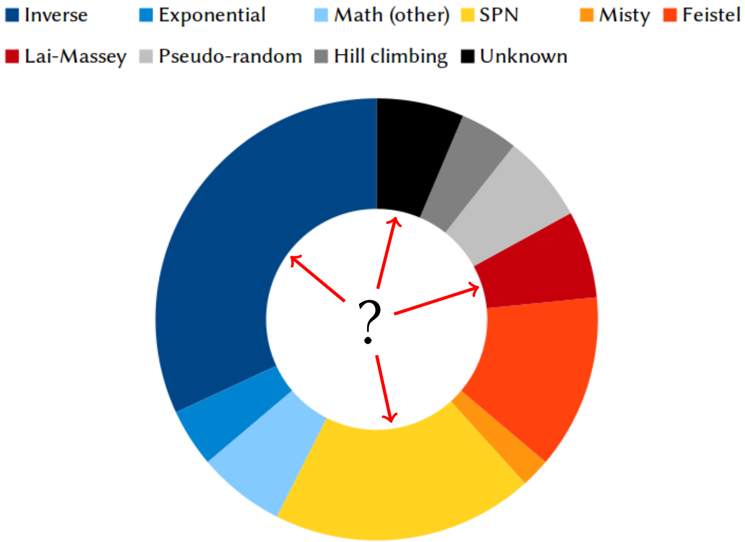
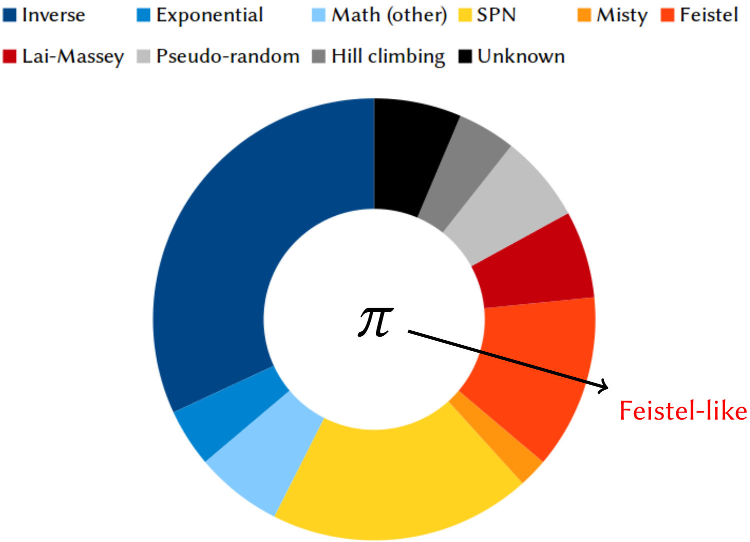
AES →
← Whirlpool
← Scream

S-Box Reverse-Engineering



Results on Kuznyechik/Streebog



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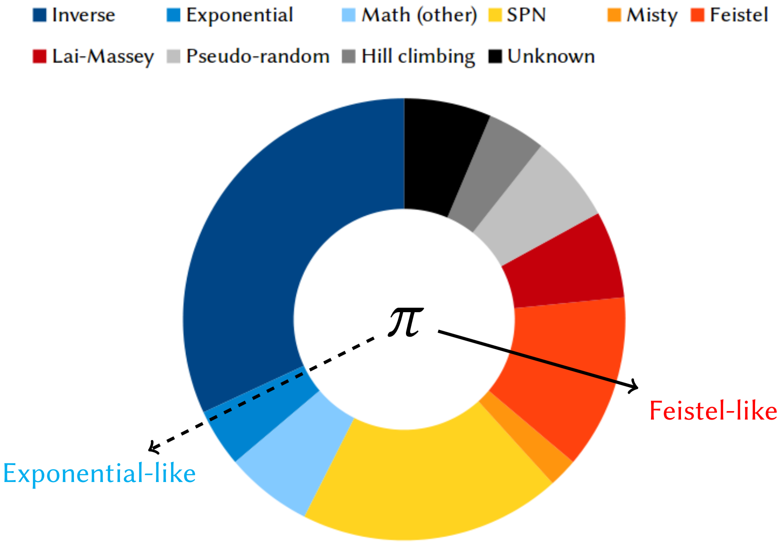
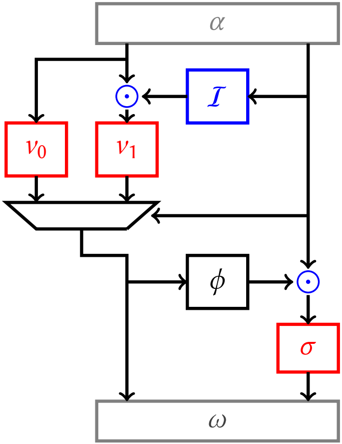


Outline

Plan

A First Decomposition of π



- From Eurocrypt'16
- α, ω: linear 8-bit permutations
- ν_0, ν_1, σ: 4-bit permutations
- ϕ: 4-bit function ($\phi(x) \neq 0$)
- $\mathcal{I}$ multiplicative inverse in $\mathbb{F}_{16}$
- $\odot$ multiplication in $\mathbb{F}_{16}$

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Decomposition Procedure Overview

1. Identify patterns in LAT;

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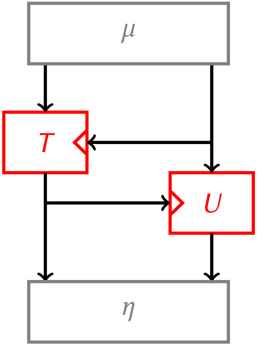

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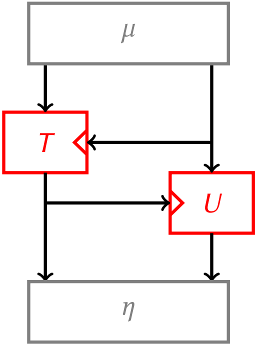

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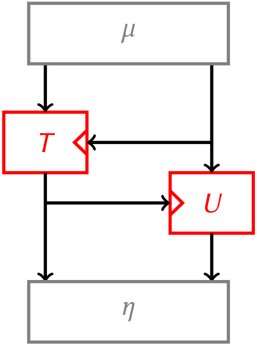
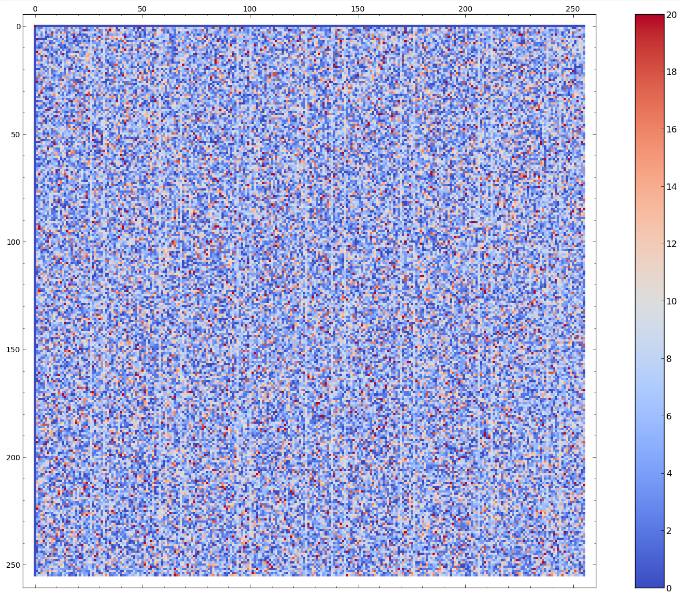
3. Decompose U, T;
4. Put it all together.



Pollock to the Rescue



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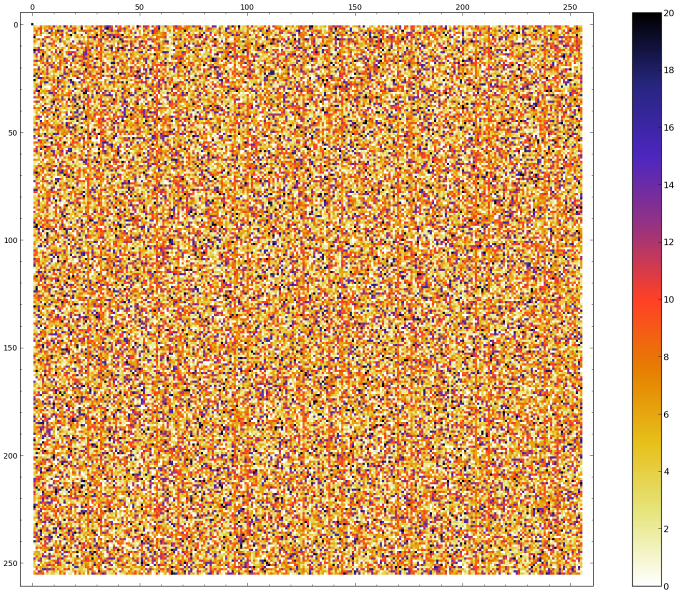
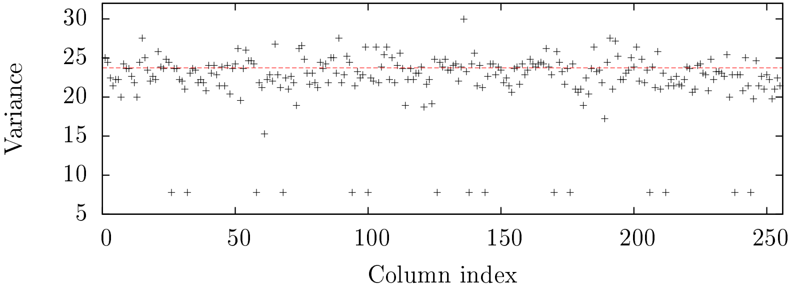


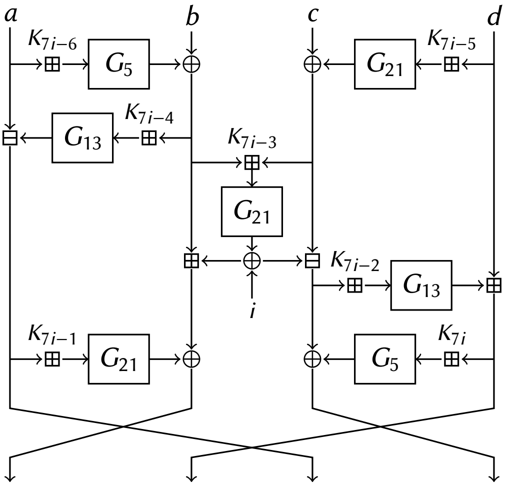
What the Lines Mean



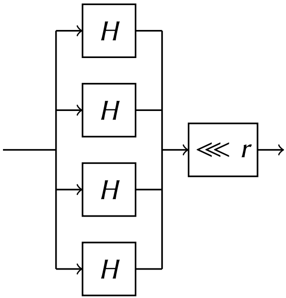
Variance of the absolute value of the coefficients in each column of the LAT of π.

Plan

Round Function of BelT

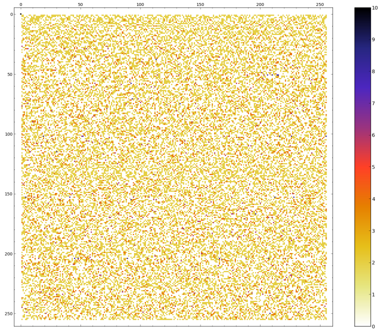


The round function of BelT.

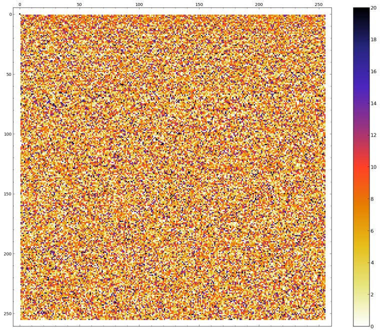


The 32-bit function G_r.

Properties of H



DDT



LAT

- max(DDT) = 8
- max(LAT) = 26
- $P[random] \leq 2^{-122}$

- Algebraic degree 7 (all coordinates)

Structure of H (1/3)

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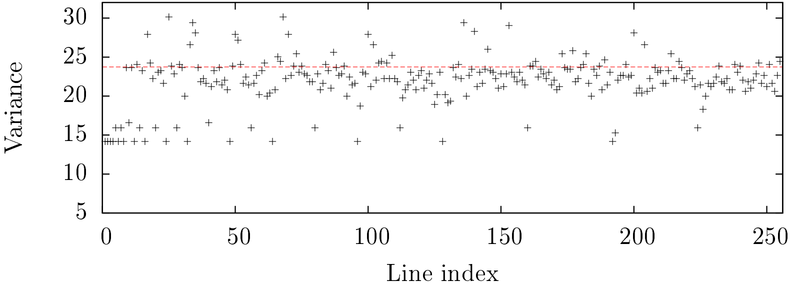
Yes!

LAT Row Variance



Variance of the absolute value of the coefficients in each row of the LAT of H.

The Actual Structure

The BelT S-Box Construction (translated)

The look-up tables of the S-Box coordinate functions were chosen as different segments of length 255 of different linear recurrences defined by the irreducible polynomial $p(\lambda)$:

$$p(\lambda) = \lambda^8 + \lambda^6 + \lambda^5 + \lambda^2 + 1.$$

Additionally, a zero element was inserted in a fixed position of each segment.

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Equivalent Pseudo-Exponential Representation

$$S \;:=\; [w^i, i < z] \;+\; [0] \;+\; [w^i, z \leq i]$$

Exponential (case $z = 0$) studied in [AA04][1]

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Properties of (Pseudo-)Exponentials

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- For exponentials, for all $a \in \mathbb{F}_2^n, r \in \mathbb{N}$:

$$\left\{\mathrm{LAT}[a, b],\ \forall b\right\} = \left\{\mathrm{LAT}[(a \lll r), b],\ \forall b\right\}$$

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- For pseudo-exponentials, for all ℓ, for $r < \log_2(z)$:

$$\left\{\text{LAT}[a, b],\ \forall b\right\} = \left\{\text{LAT}[(a \lll r), b],\ \forall b\right\}$$

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- good nonlinearity,
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Fair enough...

... but then what of π?

Plan

Exponential-Like Pattern

Observation

- $x \oplus 2^j = x \boxplus 2^j$ if $x_j = 0$ and $x \oplus 2^j = x \boxminus 2^j$ if $x_j = 1$
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$$\Longrightarrow \quad \Pr\left[w^{x \oplus 1}/w^x = w\right] = 1/2 \quad \text{and} \quad \Pr\left[w^{x \oplus 1}/w^x = w^{-1}\right] = 1/2$$

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In the case of π

Let $C = [\texttt{0x12}, \texttt{0x26}, \texttt{0x24}, \texttt{0x30}]$. Then:

$$\Pr\left[\begin{cases} \pi^{-1}(x \oplus C[i]) \ / \ \pi^{-1}(x) = w^{2^i}, \text{ or} \\ \pi^{-1}(x \oplus C[i]) \ / \ \pi^{-1}(x) = w^{-2^i} \end{cases}\right] = \frac{240}{256} .$$

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5. Find linear patterns in $\tau \circ \alpha^{-1}$;
6. Deduce better linear layer β such that $\tau \circ \beta^{-1}$ is even more structured

Structure of π^{-1}

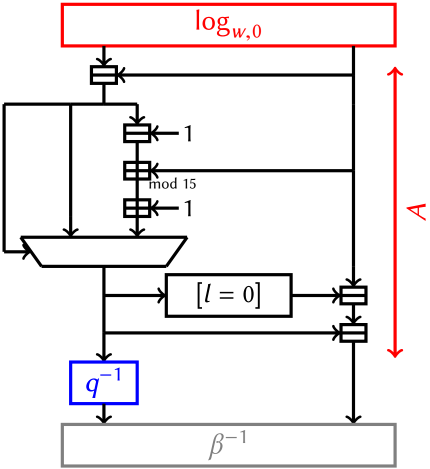
Algorithm 1 Computing the inverse of π: $y = \pi^{-1}(x)$.

$(l||r) \leftarrow \beta(x)$
$l \leftarrow q(l)$
if $l = 0$ **then**
  $z \leftarrow 17 \times ((r+1) \mod 16)$
else
  $z \leftarrow 17 \times l + r - 16$
end if
$y \leftarrow \exp_{w,0}(z)$
return y

β: 8-bit linear permutation ; q: 4-bit S-Box

$\exp_{w,0}(z) = w^z$, but $\exp_{w,0}(0) = 0$

First Decomposition of π



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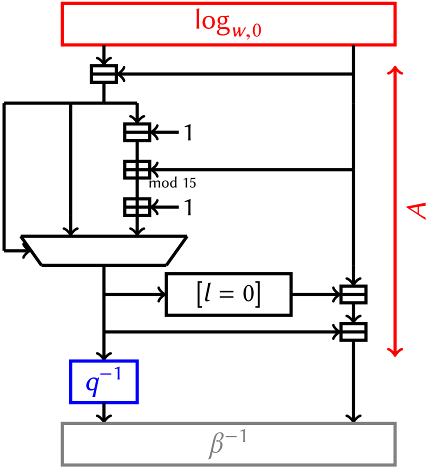
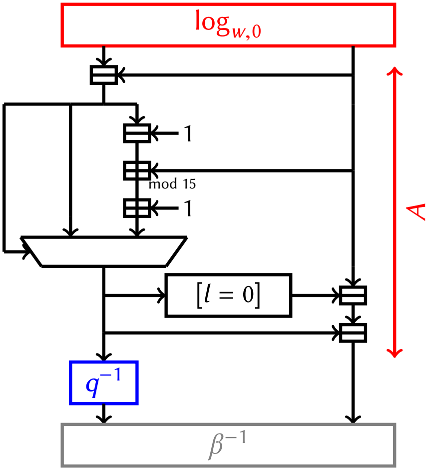


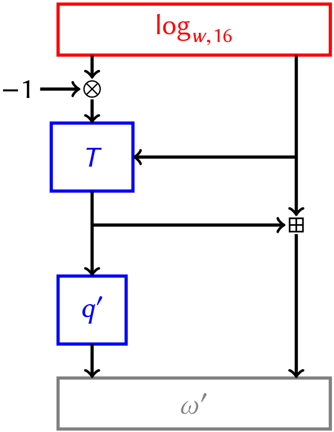
A is extremely weak...

First Decomposition of π



A is extremely weak... Can we simplify it even further using a pseudo-exponential?

A Second Decomposition of π



	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
T_0	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
T_1	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
T_2	0	1	2	3	4	5	6	7	8	9	a	b	c	d	f	e
T_3	0	1	2	3	4	5	6	7	8	9	a	b	c	f	d	e
T_4	0	1	2	3	4	5	6	7	8	9	a	b	f	c	d	e
T_5	0	1	2	3	4	5	6	7	8	9	a	f	b	c	d	e
T_6	0	1	2	3	4	5	6	7	8	9	f	a	b	c	d	e
T_7	0	1	2	3	4	5	6	7	8	f	9	a	b	c	d	e
T_8	0	1	2	3	4	5	6	7	f	8	9	a	b	c	d	e
T_9	0	1	2	3	4	5	6	f	7	8	9	a	b	c	d	e
T_a	0	1	2	3	4	5	f	6	7	8	9	a	b	c	d	e
T_b	0	1	2	3	4	f	5	6	7	8	9	a	b	c	d	e
T_c	0	1	2	3	f	4	5	6	7	8	9	a	b	c	d	e
T_d	0	1	2	f	3	4	5	6	7	8	9	a	b	c	d	e
T_e	0	1	f	2	3	4	5	6	7	8	9	a	b	c	d	e
T_f	0	f	1	2	3	4	5	6	7	8	9	a	b	c	d	e

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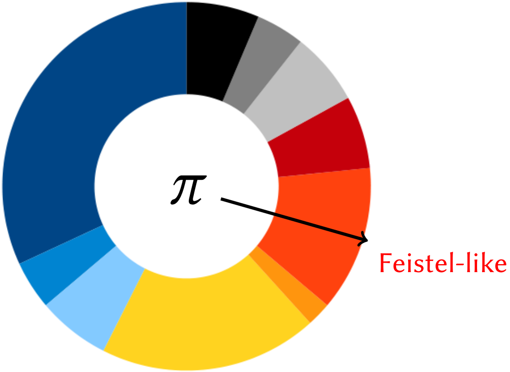
Still, $\pi^{-1} \circ \log_{w,16}$ is **differentially 128-uniform!**

- For random 8-bit permutation, $Pr[\max(\text{DDT})] = 128 \approx 2^{-346}$
- $\implies$ π is related to an exponential.

Plan

Conclusion



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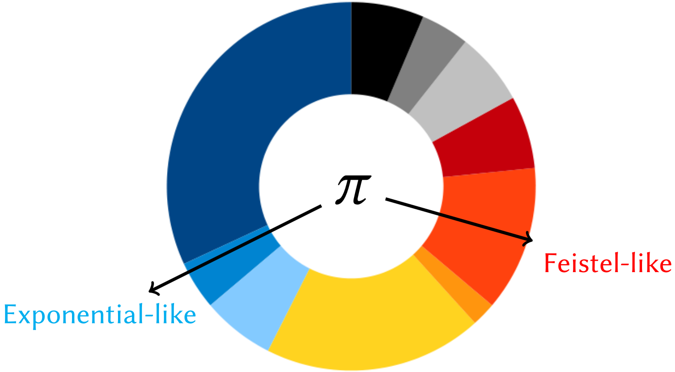

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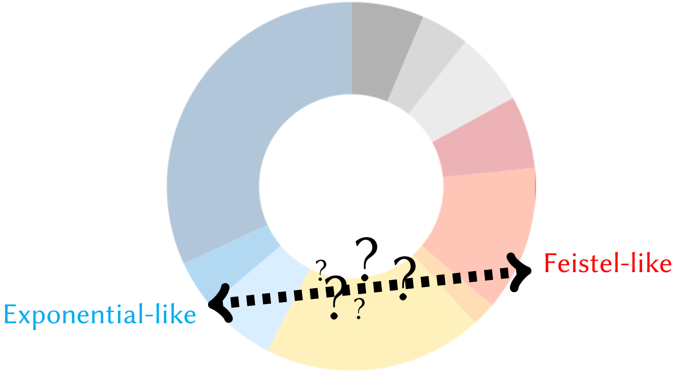

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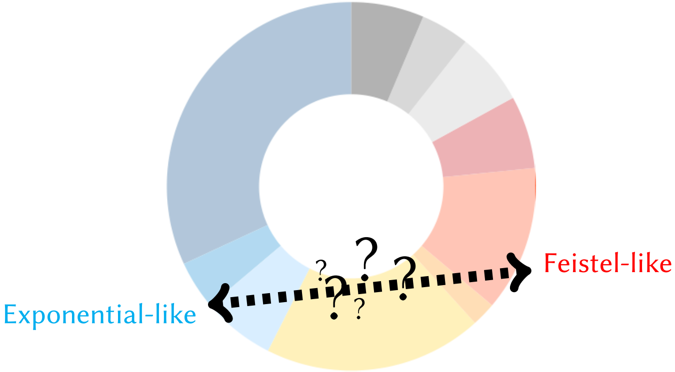

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