Radbound University

Indifferentiability of the Sponge Construction with a Restricted Number of Message Blocks

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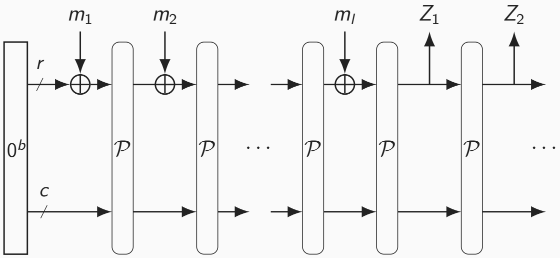
The Sponge Construction [Bertoni et al., 2007]



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- $m_1 \| \cdots \| m_l$ is the message padded into r-bit blocks

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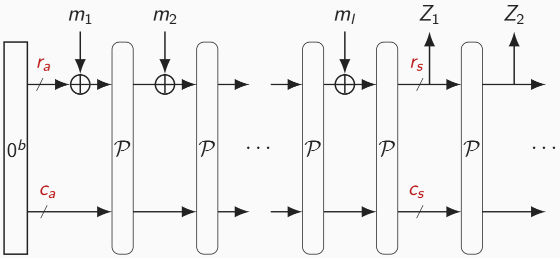

- Extendable output function
- $m_1 \| \cdots \| m_l$ is the message padded into r_a-bit blocks
- Absorb rate and squeeze rate different [Guo et al., 2011, Naito and Ohta, 2014]

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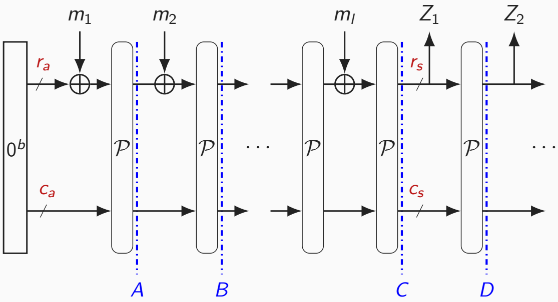
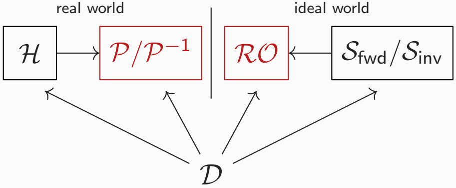
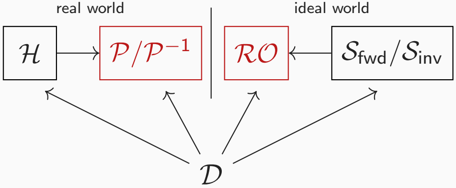


- Extendable output function
- $m_1 \| \cdots \| m_l$ is the message padded into -bit blocks
- Absorb rate and squeeze rate different [Guo et al., 2011, Naito and Ohta, 2014]
- Graph notation: $0^b \xrightarrow{m_1} A \xrightarrow{m_2} B \longrightarrow \cdots \xrightarrow{m_l} C \longrightarrow D$

Indifferentiability [Maurer et al., 2004, Coron et al., 2005]



- $(\mathcal{H}^{\mathcal{P}}, \mathcal{P})$ for a random primitive $\mathcal{P}$ should behave like a random oracle $\mathcal{RO}$ paired with a simulator $\mathcal{S}$ that maintains construction-primitive consistency
- $\mathcal{H}$ is indifferentiable from $\mathcal{RO}$ for some simulator $\mathcal{S}$ whenever any $\mathcal{D}$ can distinguish the two worlds only with a negligible probability
- This probability is usually expressed as a function of the number of queries made



- Indifferentiability advantage:

$$\mathbf{Adv}^{\mathrm{iff}}_{\mathsf{Sponge}}(q) = \max_{\mathcal{D} \text{ with } q \text{ queries}} \left| \mathbf{Pr}\left(\mathcal{D}^{\mathsf{Real}} = 1\right) - \mathbf{Pr}\left(\mathcal{D}^{\mathsf{Ideal}} = 1\right) \right|$$

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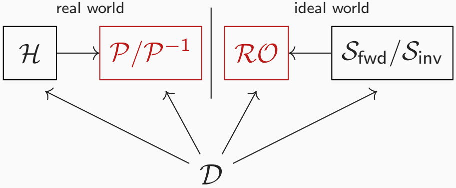

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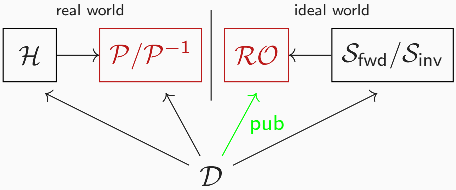
- Consider the following restriction:

$$\mathbf{Adv}^{\mathrm{R\text{-}iff}}_{\mathsf{Sponge}}(q, \ell) = \max_{\substack{\mathcal{D} \text{ with } q \text{ queries}, \\ \mathsf{pad}(M) \leq r_a \times \ell}} \left| \mathbf{Pr}\left(\mathcal{D}^{\mathsf{Real}} = 1\right) - \mathbf{Pr}\left(\mathcal{D}^{\mathsf{Ideal}} = 1\right) \right|$$

Public Indifferentiability [Yoneyama et al., 2009, Dodis et al., 2009]



- All construction queries are public $\implies$ helps the simulator to keep $\mathcal{RO}$-consistency
- Weaker model than (plain) indifferentiability: e.g., (plain) Merkle-Damgård is not indifferentiable but publicly indifferentiable [Dodis et al., 2009]
- Useful in practice, e.g., digital signature schemes

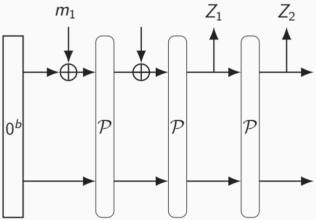
Sponge Indifferentiability

- Sponge indifferentiable with bound $\mathcal{O}\left(\frac{q^2}{2^c}\right)$ [Bertoni et al., 2008]
- Generalized sponge indifferentiable with bound $\mathcal{O}\left(\frac{q}{2^{c_a/2}}\right)$ as long as $c_s \geq c_a/2 + \log_2(c_a)$ [Naito and Ohta, 2014]

$\Longrightarrow$ At least $2^{c_a/2}$ queries to differentiate with high probability

- Tight bound: inner collisions while absorbing allow to differentiate

Collision Attack with $q \approx 2^{c_a/2}$ queries [Bertoni et al., 2011]



- Query $\mathcal{P}(m_1 \| 0^{c_a})$ for $2^{c_a/2}$ different m_1's and store them in a list L

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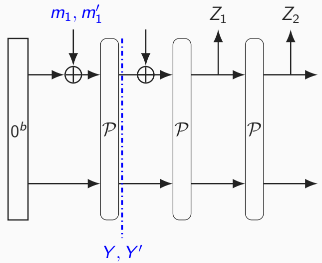

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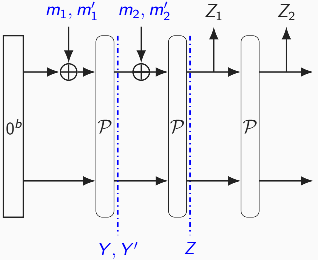

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$\Longrightarrow$ Take $m_2 = \text{outer}_{r_a}(Y)$ and $m_2' = \text{outer}_{r_a}(Y')$

$\Longrightarrow$ It gives $0^b \xrightarrow[m_1 \| m_2]{m_1' \| m_2'} Z$

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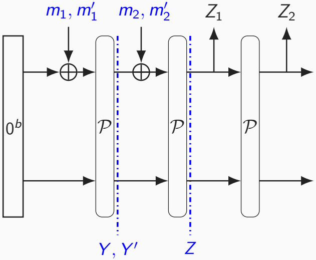

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- Requires $r_a \geq c_a/2$ and two absorb calls

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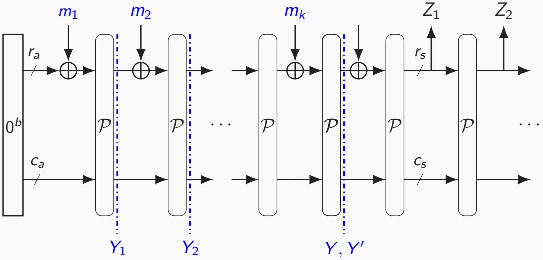

General case:

- Let $k = \lceil \frac{c_a}{2r_a} \rceil$
- One absorb round gives 2^{r_a} different states: not enough for an inner collision
- To have $2^{c_a/2}$ states (thus an inner collision w.h.p.,) need k absorb calls

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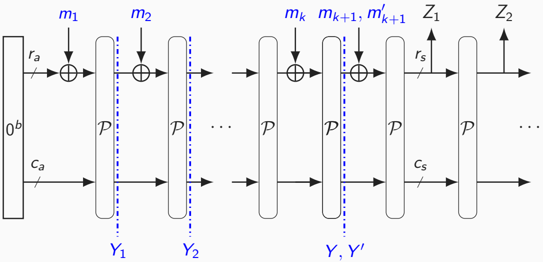

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- Need also the compensation absorb call to have a full-state collision

$\Longrightarrow$ Requires $k+1$ absorb calls

Restricted Sponge

- Consider a sponge where at most ℓ absorb calls are allowed (but an arbitrary number of blocks can be squeezed)
  - Restrictive setting
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  - Useful in e.g., password hashing, Fiat-Shamir transform
- When $\ell < k + 1$, the collision (thus differentiability) attack on the sponge does not apply anymore
- One full-state collision attack in $2^{b-\ell \times r_a}$ queries:
  1. Make all $\ell - 1$ first absorb call queries to obtain $\left(0^b \xrightarrow{M_i} Y_i\right)_i$
  2. Compute with primitive queries $0^b \xrightarrow{M_1} Y_1 \to N_1 \cdots \to N_{2^{b-\ell \times r_a}}$
  3. $2^{(\ell-1) \times r_a}$ Y_i states and $2^{b-\ell \times r_a}$ N_i states $\implies$ inner collision between some Y_i and N_j happens with high probability
  4. Use the last absorb call on Y_i to obtain a full state collision

Tightness of Indifferentiability With a Restricted Sponge

- Attack has a cost of $2^{b-\ell\times r_a}$ while indifferentiability of the sponge guarantees security up to $\approx 2^{c_a/2}$ queries

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- Contribution of this work:

$$\mathbf{Adv}^{\text{R-iff}}_{\mathsf{Sponge}}(q,\ell) = \mathcal{O}\left(\frac{q}{2^{c_s}} + \frac{q^2}{2^b} + \min\left\{\frac{q^2}{2^{c_a}}, \frac{q}{2^{b-\ell\times r_a}}\right\}\right)$$

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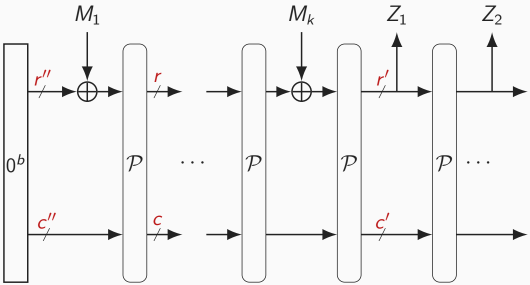

- When $\ell = 1$ the bound is already captured by an indifferentiability result from Naito and Ohta: set $r = 0, r' = r_s, r'' = r_a$
- New results whenever $1 < \ell < \lceil \frac{c_a}{2r_a} \rceil + 1$

Notation

- Define `AbsorbPath` as

$$\texttt{AbsorbPath} = \left\{0^b\right\} \cup \left\{ Y \mid \exists 0^b \xrightarrow{m_1\|\cdots\|m_l} Y \text{ with } l < \ell \right\}$$

$\Longrightarrow$ `AbsorbPath` contains the rooted nodes where absorption of a message block is still possible

- Remark: $|\texttt{AbsorbPath}| \le \min\left\{q+1, 2 \times 2^{(\ell-1)\times r_a}\right\}$

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- Remark: $|\texttt{AbsorbPath}| \leq \min\left\{q+1, 2 \times 2^{(\ell-1)\times r_a}\right\}$
- $0^b \xrightarrow{m_1} A_1 \xrightarrow{m_2} \cdots \xrightarrow{m_l} A_l \longrightarrow S_1 \longrightarrow \cdots \longrightarrow S_n$ is a valid path whenever $l \leq \ell$

The Simulator Used

$\mathcal{S} = (\mathcal{S}_{\mathsf{fwd}}, \mathcal{S}_{\mathsf{inv}})$, similar to the one used in indifferentiability of sponge proof [Bertoni et al., 2008]:

- $\mathcal{S}$ keeps track of the graph construction
- $\mathcal{S}_{\mathsf{inv}}$ returns random elements
- On query with input X, $\mathcal{S}_{\mathsf{fwd}}$ keeps $\mathcal{RO}$-consistency whenever X appears in a valid path
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- $\mathcal{S}$ behaves like a two-sided RF
- For public indifferentiability: build $\mathcal{S}'$ which additionally relays to $\mathcal{S}$ all primitive queries associated to the construction queries

World Decomposition

$$
\begin{tikzcd}
\text{Ideal World} & & & \text{Real World} & \\
\boxed{\mathcal{RO}} & \boxed{\mathcal{S}_{\mathsf{fwd}}/\mathcal{S}_{\mathsf{inv}}} \arrow[l] & & \boxed{\mathcal{H}} \arrow[r] & \boxed{\mathcal{P}/\mathcal{P}^{-1}} \\
& & \mathcal{D} \arrow[ull, bend left] \arrow[ul, bend left] \arrow[ur, bend right] \arrow[urr, bend right] & &
\end{tikzcd}
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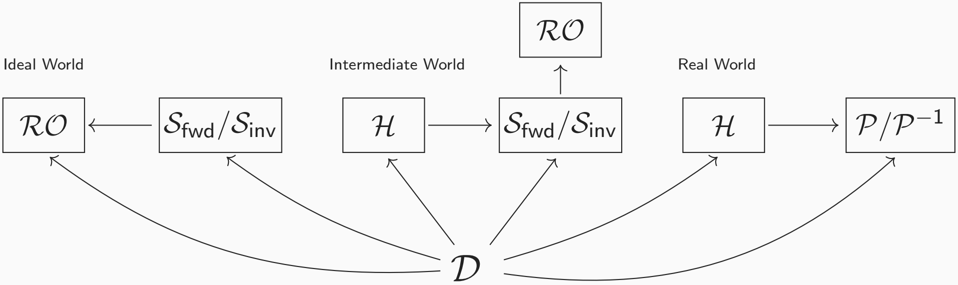

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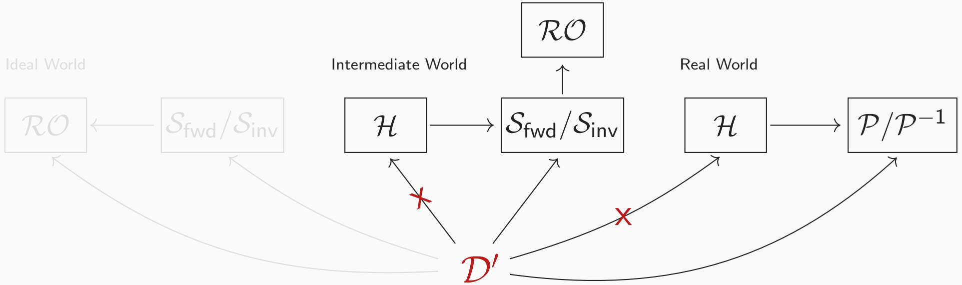

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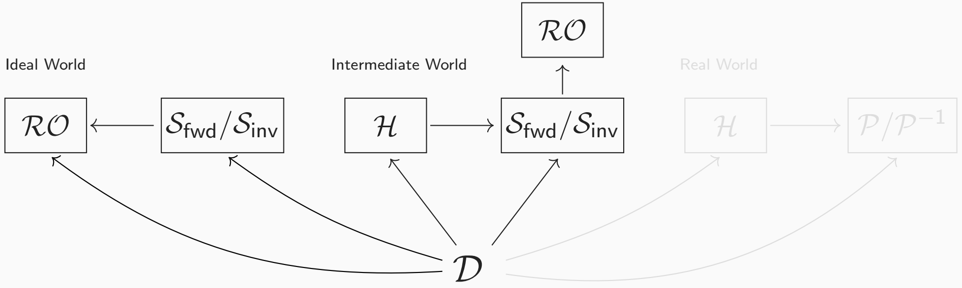

- One Intermediate World is introduced to facilitate the analysis
- Intermediate versus Real: construction queries can be transformed into primitive queries $\implies$ PRP/PRF switching lemma
- Ideal versus Intermediate: consistency of the simulator with respect to $\mathcal{RO}$ and extra queries to $\mathcal{S}$ in Intermediate World $\implies$ identical until **BAD**

Ideal versus Intermediate: Bad Events

$$\mathbf{Adv}^{\text{R-iff}}_{\text{Sponge}}(q,\ell) = \mathcal{O}\left(\frac{q}{2^{c_s}} + \frac{q^2}{2^b} + \min\left\{\frac{q^2}{2^{c_a}}, \frac{q}{2^{b-\ell\times r_a}}\right\}\right)$$

- **GUESS**: (only in Intermediate World) adversary guesses an intermediate state generated from construction queries without having made the primitive queries

  To do that, it can guess:

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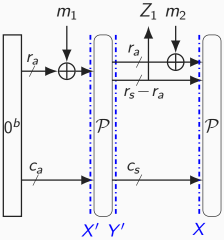
  GUESS does not apply in public indifferentiability



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- **CONNECT**: $Y_i = X_j$ or $X_i = Y_j$ for some $j < i$

Tightness of the bound

- Remember that

$$\mathbf{Adv}^{\text{R-iff}}_{\text{Sponge}}(q,\ell) = \mathcal{O}\left(\frac{q}{2^{c_s}} + \frac{q^2}{2^b} + \min\left\{\frac{q^2}{2^{c_a}}, \frac{q}{2^{b-\ell\times r_a}}\right\}\right)$$

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- Inner collision attack has a cost of $\approx \max\left\{2^{c_a/2}; 2^{b-\ell\times r_a}\right\}$ queries
- What about the others terms?
  - 2^{c_s} queries: adversary can try all inner parts
  - $2^{b/2}$ queries: adversary can set **CONNECT**

Application

- Ascon-hash
  - $b = 320, c = 256, r = 64$
  - Unrestricted sponge: 128 bits of security
  - Sponge with input messages of at most 127 bits: 160 bits of security
- Photon Beetle-Hash or T-Quark
  - $b = 256, c = 224, r = 32$
  - Unrestricted sponge: 112 bits of security
  - Sponge with input messages of at most 127 bits: 128 bits of security
- To maximize security and absorbing rate, the best parameter choice is $\ell = 1, c_a = r_a = b/2$

Conclusion

- Proved a tight indifferentiability bound for the sponge construction when the number of message blocks is restricted
- It gives a better security bound when less than $\lceil \frac{c_a}{2r_a} \rceil + 1$ blocks are absorbed

Thank you for your attention!

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